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Adjusting to the Euro*

Gabriel Fagan† and Vítor Gaspar‡

13 December 2006

Abstract

In this paper we argue that, for a group of converging economies of the European Union, participation in the euro area has been associated with easier access to financing by domestic economic agents. Easier access to financing was a significant impulse leading to a sharp increase in households' expenditures and a corresponding fall in the savings ratio. Increased expenditure was associated with current account deficits, a sharp fall in the net foreign asset position and an increase in the households' indebtedness. At the same time there was a sizeable increase in the real exchange rate. In this paper, we show that it is possible to obtain all these qualitative features of adjustment using a simple analytical model of intertemporal equilibrium. Specifically, we consider a simple endowment economy with traded and non-traded goods populated by Blanchard-Yaari households. We also argue that the consideration of external habit formation improves the model's ability to mimic short to medium term adjustment dynamics while, at the same time, improving the plausibility of steady state effects.

JEL Classification: F36, E21, F32

Keywords: euro area, interest rate convergence, overlapping generations model

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NON-TECHNICAL SUMMARY

Participation in a monetary union entails manifold and complex consequences. Even disregarding the important fact that economic and political integration are co-determined and co-evolutionary, participation in the euro area affects trade and financial integration, business cycle synchronization and the patterns of specialization, and also the institutions, structure and behavior in product and labor markets. The issue is of particular importance in Europe, where the set of countries that have adopted the euro already includes twelve Member States. Moreover, macroeconomic adjustment to the prospect of participation in the euro area is a topical issue for the new Member States of the European Union.

In this paper, we will argue that, for countries like Greece, Spain, Ireland, Italy and Portugal (hereafter referred to as 'converging countries'), one important aspect of the process of adjustment to participation in the euro area was associated with the convergence of short and long term interest rates to the relatively low levels prevailing in Germany. From the viewpoint of these countries, participation in the euro area entails easier access to international financial markets and is also associated with financial liberalization and financial integration. There are two main reasons to focus on the financial dimension of euro area participation. First, compared with other effects from euro area participation (nominal), interest rate convergence and financial integration are easy to document on the basis of available statistical information and both happened relatively fast. Second, the available evidence indicates that the effects are large and significant.

In the paper, we do two things. First, we document the stylized facts regarding the macroeconomic effects of interest rate convergence on the converging countries. Second, we follow Blanchard and Giavazzi (2002) and Fagan and Gaspar (2005) and examine the ability of simple macroeconomic models to explain the observed patterns of adjustment.

As regards the stylized facts, we find that the convergence in interest rates has been associated with a sharp increase in households? expenditures and a pronounced increase in households? debt in the converging countries. The expansion in expenditure does not seem to have been associated with noticeable effects on output or with sizeable effects on private sector productive investment. Instead, it has been associated with deterioration in the current account deficit and with the accumulation of sizeable negative net foreign asset positions. At the same time, the converging countries recorded inflation differentials which, under exchange rate stability followed by the adoption of the euro, implied significant real appreciation and loss of competitiveness, according to standard indicators.

The model we develop relies on a standard Blanchard-Yaari endowment

economy setup with traded and non-traded goods. The exercise is in the spirit of Obstfeld and Rogoff (2004), who used an endowment economy framework, with traded and non-traded goods, to discuss the real exchange rate implications of changing the geographical patterns of world expenditure. Our setting allows us to study the effects on the real exchange rate of changing expenditures patterns over time. The simple Blanchard-Yaari model is able to account qualitatively for all the stylized facts reported above. However, with standard time-separable preferences, expenditure jumps up on impact and, immediately thereafter, its growth rate goes below baseline. Moreover, the steady state effects on the net foreign asset position seem implausibly large. So, finally, we go on to show that the introduction of external habit formation solves both problems. The initial build up in expenditure is gradual and the size of effects on the steady state is much diminished. Therefore, we conclude that a simple Blanchard-Yaari model of a small, open, endowment economy, with habit formation and traded and non-traded goods, goes a long way in explaining the adjustment of the converging countries to the euro.

1 INTRODUCTION

Participation in a monetary union entails manifold and complex consequences. The issue is of particular importance in Europe, where twelve EU Member States[1] have already adopted the euro as their single currency. Even disregarding the important fact that economic and political integration are co-determined and co-evolutionary, participation in the euro area affects trade and financial integration, business cycle synchronization and the patterns of specialization. It also affects institutions, and the structure and behavior of product and labor markets (see Mongelli and Vega (2006) and references therein for comprehensive surveys of the relevant issues and available evidence). Moreover, macroeconomic adjustment to the prospect of participation in the euro area is a topical issue for the new Member States which joined the European Union in 2004.[2]

In this paper, we will argue that, for countries like Greece, Spain, Ireland, Italy and Portugal (which we will refer to as converging countries) an important aspect of the process of adjustment to participation in the euro area is associated with easier access to finance for domestic economic agents. The most evident aspect of such a process is the convergence of short and long term interest rates to the relatively low levels prevailing in Germany and other low interest rates European countries. Nevertheless, it is clear that, from the viewpoint of converging countries, participation in the euro area entails easier access to international financial markets and is also associated with the broader processes of financial liberalization and financial integration. In our view, the focus on the financial dimension of participation in the euro area can be justified by two considerations. First, compared with other effects from euro area participation interest rate convergence and financial integration are easy to document on the basis of available statistical information and happened relatively fast. Second, from a macroeconomic point of view, the effects are likely to be large and significant. As we document below, economic trends in the converging countries were significantly different from those registered in countries like Germany or countries more closely integrated with Germany like Belgium, The Netherlands, Finland, France, Luxembourg and Austria. In particular, we find that the convergence of interest rates has been associated with a boom in household expenditures accompanied by a sizeable build-up in household debt. This higher expenditure appears not to have had marked effects on output but instead to have been reflected in a deterioration in the current account balance and in the net foreign asset position and to have been accompanied by a notable appreciation of these countries intra euro area real exchange rates. In an earlier paper (Fagan and Gaspar (2005)), we examined the effects of a permanent fall in interest rates

[1] Belgium, Greece, Spain, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Austria, Portugal and Finland. In March 2006, Slovenia and Lithuania requested the European Commission and the European Central Bank, the preparation of a Convergence Report, examining whether a high degree of sustainable convergence has already been achieved, enabling their participating in the euro area. Following the required procedures it was found that Slovenia satisfied the required criteria. Slovenia will participate in the euro area from January 2007.

[2] Czech Republic, Estonia, Cyprus, Hungary, Latvia, Lithuania, Malta, Poland, Slovenia and Slovakia

in the context of a dynamic general equilibrium model of a small open economy, with traded and non-traded sectors and a number of frictions, such as a adjusment costs for investment and sticky prices. Again, the results indicated a notable deterioration in the current account balance following the initial impulse. The response of output to the shock was found to be slow and gradual while the dynamics of the current account and prices were dominated by the pattern of consumption.

Blanchard and Giavazzi (2002) explore the implications of entry into the euro area for countries such as Portugal and Greece via financial and goods market integration. For this purpose, they use a two-period, two country model. They modelled financial integration into the Euro Area as a decline in the exogenous risk premium in domestic interest rates, in the high interest rate country. Their main finding is that the fall in interest rates is associated with an increase in the current account deficit in borrowing countries, and with a wider cross-country dispersion of current account balances. While the framework provides powerful insights, a problem with the two-period setup is that it provides no information on the crucial aspect of the dynamics of the adjustment path, e.g. the persistence of current account deficits and the duration of convergence towards steady state equilibrium.

Against this background, we argue that standard intertemporal macroeconomic frameworks, even abstracting from the production side of the economy, can go a long way in explaining the key features of this adjustment process. Specifically, we consider the combination of a simple endowment economy models, with a household sector, modelled along the lines of Blanchard (1985) and Yaari (1965). Such a model is uniquely suited to illustrate the importance of intertemporal expenditure effects which we believe are crucial for the case at hand. We consider the case with two goods (traded and non-traded). We will argue that such simple variants are already capable of capturing most salient, qualitative features of adjustment, present in much more complicated intertemporal models. Specifically, in the two goods endowment economy, the fall in the opportunity cost of external financing leads to an initial increase in expenditure followed by slow adjustment to the new lower steady state as foreign debt accumulates. In our model, the real exchange rate (defined as the relative price of non-tradable goods) follows exactly the path of aggregate expenditure. In other words, it increases initially (there is real appreciation) followed by a gradual decline to a level lower than in the initial steady state.

The analysis we perform for the endowment economy with non-traded goods is in the spirit of Obstfeld and Rogoff (2004). They also use a endowment economy framework, with traded and non-traded goods to discuss the real exchange rate implications of changing expenditure patterns. They focus on the implications of relocating expenditure in the global economy. From changing geographical patterns of expenditure implications for real exchange rates follow. In this paper, we focus instead on the implications for real exchange rates from different intertemporal patterns of expenditure, endogenously generated by changes in a country's access to external financing. In our analysis, we focus on the case of the small open economy which enters a monetary union. The economy faces an

exogenously given traded-goods price (which for convenience we will keep constant) and an interest rate which is determined by the "world " (or, more specifically, area-wide) interest rate plus an exogenous risk premium. The shock resulting from financial integration into the monetary union is modelled - following Blanchard and Giavazzi - as a reduction in this exogenous risk premium.

The basic version of the model which we employ has the undesirable feature that, following the shock of entry into the monetary union, consumption jumps immediately and by a substantial magnitude before slowly converging to its new steady state. This property is at odds with the rather gradual pattern of adjustment which can be seen in the data. The pattern observed in the data might be explained in a number of ways. First, as we document below, expectations regarding the creation of the monetary union and the participation of particular countries evolved during the course of the 1990s in response to political and economic developments. Hence, modelling EMU entry as a sharp one-off event as done in this paper is a simplification. Second, for the countries we are examining, EMU represented a new regime, so that agents may have required time to learn about the new environment, a phenomenon which would be expected to lead to a gradual adjustment on the part of agents. Third, agents's expenditures may in any case be subject to an inherent sluggishness, reflecting various types of adjustment costs or habit persistence.

In order to obtain a realistic pattern of adjustment in a convenient and tractable way we extend the basic model by incorporating habit formation into the consumer's optimisation problem. Here we take a variant of the simplest version of habit formation from Abel (1990), Campbell and Cochrane (1999) and Carroll, Overland, and Weil (2000). In our setup, habits are "external " or of the "keeping up with the Joneses " variety, and depend on average consumption in the economy as a whole. It is documented in the literature that habit formation not only helps to explain the persistence in consumption it also contributes to sorting out a number of important empirical finance puzzles. Moreover, available empirical evidence (for example, Gruber (2004)) suggests that the addition of habit formation leads to a significant improvements in the empirical performance of intertertemporal current account models such as the one employed in this paper. While, following the pioneering work of Mansoorian (1993), the inclusion of habit formation in infinitely lived agent models is straightforward and widely used, its incorporation into a Blanchard-Yaari setup involves a number of additional complexities and is rarely employed in the literature. Examples are Obstfeld (1992) and Velculescu (2006) who incorporate habit formation into one-sector open economy models. The introduction of habit formation into our model changes the results in two important ways: First, and as expected, we obtain a gradual build up of aggregate expenditure in the short to medium term. Second, and more surprisingly, we derive smaller effects in steady state. The latter effect is also very helpful as the steady state predictions from the simpler formulation look implausibly large.

The paper is organized as follows. In Section 2, we review some stylized facts in order to motivate our analysis. In Section 3, we present our model. In Section 4, we present

numerical simulations for the case with traded and non-traded goods case, contrasting results with and without habit formation. Section 5 concludes.

2 SOME STYLISED FACTS.

The creation of the monetary union in Europe in 1999 was preceded by a process of nominal convergence among the participating countries. In 1996, for example, nominal long term interest rates ranged from a high of 14.5 percent in Greece to 4.5 percent in Germany. Today spreads between the highest and lowest Government bond yields is of the order of 30 basis points. Similarly, the highest-lowest spread in short-term interest rates amounted to 10.5 percentage points in 1996. Currently, this spread is effectively zero. Of course, a significant component of the convergence in nominal interest rates across countries reflected a convergence of expected inflation across countries. However, the evidence shows that real interest rates were also significantly affected. While reliable measures of real long term rates at the country level are not available for this period, the evidence from measures of real short-term interest rates (nominal rates minus actual inflation) points to a significant convergence in the period leading to monetary union although much less marked than in the case of nominal rates. In 1996, for example, real short term rates in excess of 4% were common in a number of current EMU member countries while, for other so-called core countries (or, in our prefered wording reference countries) real short term interest rates between 1 and 2 percent were the order of the day. After the start of the single monetary policy real interest rate differentials reflected differences in inflation among participating Member States.

This evidence thus suggests those countries which experienced a decline in their interest rates as a result of the move to EMU, experienced a significant and permanent reduction in the financing costs available to their domestic economic agents. This process happened rather quickly and also appears to have contained a large unanticipated element. For example, according to various EMU calculators, the probability that some of these countries would enter EMU in 1999 was judged by markets to be significantly below 50% as late as 1996 (see for example, Favero, Giavazzi, Iacone, and Tabellini (2000) and Bates (1999)).

What were the economic effects of the decline in interest rates on the countries concerned? A full country-by-country account would need to reflect, in a comprehensive way, the various shocks and country-specificities affecting the different economies over this period in order to disentangle the separate contribution of interest rate convergence from other shocks. One possible approach would be to carry out counterfactual simulations using macromodels of individual countries A problem with this approach is that the results are heavily dependent on the model specification employed (Berben, Locarno, Morgan, and Valles (2004)). An alternative approach would be to attempt to use country specific VARs to identify the impact of the interest rate shock. However, this approach too has a

number of limitations. VAR results are often sensitive to the chosen identification scheme and, moreover, it is not clear that it would be possible to identify shocks of the kind we are interested in - permanent changes in financing costs - using VAR methodologies. Moreover, given the unique nature of the EMU experiment, it is not at all clear that empirical estimates based on average behaviour over past data would give an accurate picture of the impacts involved.

In the light of these difficulties, we follow an alternative approach. Specifically, we rely on a more informal event study method which aims at identifying some broad stylized facts regarding the impact of interest rate convergence. We start by separating the euro area countries into two groups on the basis of the relative nominal and real interest rates prevailing in the mid 1990s. The first group comprises low interest rate countries Germany, Belgium, France, the Netherlands and Austria[3]. In common with the usual parlance, we henceforth denote this group as core countries. The second group comprises countries which, initially, had relatively high interest rates: Spain, Ireland, Italy and Portugal[4]. We call this group the converging countries. We do not include Greece in this latter group since it entered the euro area only in 2001, some two years later than the other countries[5]. With a view to identifying pertinent general features of the data, and as far as possible reducing the effects of country-specific noise, we calculate, for each economic variable, a pair of time series based on arithmetic (equally weighted) averages within the groups. Then, with a view to controlling for the possible effects of common shocks across all countries, we compare the differences across these averaged country groups. Thus, the analysis below will be based on the analysis of cross-group differences in the behaviour of the variables of interest. Clearly, both the cross-sectional and time dimensions of the data are too limited to allow for robust statistical analysis. Nonetheless, we believe that taking this perspective on the data does yield some useful and novel insights into the effects of the interest rate convergence process. Details of the data sources are presented in the Appendix 1.

On the basis of the information derived from these calculations, the following features of the data are highlighted as key stylized facts:

First, and unsurprisingly, there was clearly a convergence of nominal and real interest rates. Charts 1 and 2 show respectively the evolution of nominal short-term and long-term interest rates in the two country groups. Convergence is evident. As regards real interest rates, there are no reliable measures of long-term real interest rates for the countries concerned. We can however easily calculate real ex-post short term interest rates. The relevant data is shown in Chart 3. Strong convergence in real rates across the two country groups occurred up to 1999 with the average differential close to zero compared to around

[3]We do not include Luxembourg in our analysis due to difficulties with data availability.

[4]We do not include Finland in either group. This reflects the fact that it only joined the EU in 1994 and the evolution of its economy over part of the period under review is heavily distorted by special factors (such as the recovery from the sharp recession recorded in the early 1990s).

[5]We have replicated the analysis including Greece in the converging group. We find that the conclusions derived in this section are not sensitive to the exclusion of Greece. Details are available on request from the authors.

290 basis points in 1996. With nominal short rates identical across countries following the move to monetary union, subsequent real interest rate differentials reflected inflation differentials. As a consequence, real interest rates in the converging countries fell below those in the core, and from 2002 on, turned negative see(ECB (2003) for a further analysis of this phenomenon). In the analytical exercises, carried out in the next two sections, the fall in interest rates will be the impulse trigering the subsequent adjustment of the economy.

We may summarize these remarks as our first stylized fact:

(1) As part of the process of adjustment to participation in the euro area, a number of converging economies benefited from significantly lower nominal and real interest rates, with the former converging to those of the other participants in the euro area.

Second, a notable feature of the data is the marked difference across country groups in the behaviour of household expenditures (consumption plus housing investment). The converging countries experienced a boom in household expenditures. This was reflected a more rapid decline in the household savings ratio (Chart 4) in the converging group. Housing investment (Chart 5) also boomed, rising as a share of GDP by 4 percentage points. In contrast, as a percentage of GDP, housing investment fell somewhat over this period in the reference countries.

(2) Household expenditures increased significantly leading to a fall in households' savings ratio.

Third, related to the previous point, there was a sharp rise in household indebtedness in the converging countries (Chart 6). Between 1995 and 2004, the ratio of household debt to GDP doubled whereas in the reference group the increase was much more muted. It is interesting to note that initially, the level of debt to GDP in the converging countries was well below the level in the reference group. The resulting difference in credit growth thus lead to a convergence in debt levels over the period. (see ECB (2005a) for further analysis).

(3) Credit to households grew strongly leading to a sharp increase in households' debt, converging to the levels prevailing in the reference countries.

Non-housing investment also rose in converging countries altough its increase was less marked than in the case of housing investment. Upto 2000, non-housing investment rose in both country groups, driven mainly by cyclical factors, with the rise being more rapid in the converging group. Thereafter, this form of investment declined as a share of GDP at a broadly comparable rate in both groups. (Chart 7).

(4) Non-housing investment also rose in the converging countries relative to the core group, albeit less notably than housing investment.

Turning to output, over the whole period, GDP growth was around 1 percentage point higher in the converging countries. Surprisingly growth differentials tended to narrow (see

Chart 8) to levels which were low by historical standards. This conclusion is confirmed by the comprehensive review of developments in output growth across euro area countries contained in Benalal, del Hoyo, Pierluigi, and Vidalis (2006).

(5) Perhaps surprisingly, pronounced changes in GDP growth differentials are not a prominent feature of the data.

With relative output growth stable or declining and domestic demand rising vigorously in relative terms, there were notable differences in the behaviour of the current account balances and the evolution of net foreign assets across the two groups of countries (Charts 9 and 10). Over the whole period, the average current account balance was basically stable at a level of around 2 percent of GDP in the reference countries. In sharp contrast, the current account balance in the converging countries, declined steadily through the period ending the period with a deficit of around 5% of GDP. The increased dispersion current account balances across euro area, and the associated weakening of standard Feldstein-Horioka correlations, has been highlighted by Blanchard and Giavazzi (2002) as one of the key macroeconomic effects of the increased financial integration resulting from the creation of the euro.

This pattern in current account balances is also reflected in the evolution of net foreign assets. According to data presented in Lane and Milesi-Ferreti (2006) the ratio of net foreign assets to GDP in the reference countries increased slightly over the period 1995-2004. In contrast, in the converging countries, reflecting cumulated current accounted deficits, the net foreign asset ratio declined by over 30 GDP percentage points (see Chart 9).

(6) Converging countries current accounts turned into a large deficit position implying the accumulation of a substantial negative net foreign asset position during the period.

A striking feature of the data is the sharp and continuing appreciation of the real exchange rate of the converging countries vis-à-vis the core group. Our measure of the real exchange rate is based on Eurostat comparative consumer price indices shown in Chart 12. is relative consumer prices in the converging versus the core group. This is based on Eurostat data on comparative price levels. On this measure, the real exchange rate in 2005 was 20% higher than its level in 1995. By definition, this appreciation reflected a persistent inflation differential between the two country groups (Chart 11). Inflation differentials in the euro area have been the subject of extensive study by both researchers and official organisations. The existing literature suggests that a number of factors are at play, including country-specific shocks, the effects of integration of goods markets resulting in price convergence, Balassa-Samuelson effects, cyclical divergences, inflation persistence and structural differences across economies which result in differential responses to external shocks (see ECB (2003) for a review of the various factors). However, the available evidence also suggests that interest rate convergence has played a prominent role (ECB (2005b)).

(7) Converging countries recorded sharp real appreciation.

Before summarising our findings, it is useful to look in more detail at one of the most striking features, and one which will play a central role in the analsis below, namely the relative behaviour of the current account balances and particularly the sharp deterioration in the current account of the converging countries. A useful way of looking at this question is to examine the sectoral balances which constitute the counterparts of the current account. The standard national accounts identity which links the current account to savings and investment

$$CA = S - I$$

may be further decomposed into

$$CA = (T - G) + (S_H - I_H) + (S_c - I_C)$$

This tells us that the current account surplus may be accounted for by three counterparts. The first is net lending by the government $(G - T)$, or, in more usual parlance, the fiscal deficit. The second element is net lending by the household sector $(S_H - I_H)$, which is the difference between gross savings by households and their investment expenditures (mainly on housing). The remaining component - which we calculate as a residual - is net lending by firms, defined in an analogous manner.

Chart 13 provides data on this decomposition for our two country groups. A first striking feature is that there was little difference across the country groups in the average fiscal balances which follow paths which are close to each other over the whole period under review. The different paths of the current balance are thus entirely reflected in differences in net lending of the private sector (both households and firms), with each component contributing to the relative deterioration of the current account in the converging countries.

At this stage, we summarise the main findings as follows. The process of interest rate convergence was accompanied by a boom in final expenditures of households (reflecting both standard consumption expenditure and housing investment) in the converging countries. This was accompanied by a sharp rise in the household debt ratio. Non-housing investment also increased in relative terms but the differences across country groups were not as sizeable. In contast, fiscal balances were remarkably similar across the two groups of countries. Overall output growth differentials, however, did not increase. The boom in domestic expenditure fueled by credit growth triggered a deterioration the current account balance and a build up of foreign debt. In addition, the converging countries experienced a sizeable real appreciation vis-à-vis the core group. Our evidence thus confirms the analysis of Lane (2006) who points out that the convergence of interest rates "generated rapid growth in lending and local housing booms in the favoured countries, with the sharp increase in demand contributing to inflationary pressures".

Is it possible to explain these facts using an analytical, intertemporal general equilibrium model? Given that changes in overall economic activity do not appear to have been a dominant element in the adjustment to lower interest rates in the converging countries, we believe it is worthwhile to look at simple endowment economy models. Hence, the question we will try to answer in the remainder of the paper is: given improved access to financing in converging countries reflected in lower interest rates facing agents, is it possible to use an intertemporal, endowment economy model to explain facts two, three, six and seven (as stated above)? We will find that the answer is a qualified yes.

3 BLANCHARD-YAARI HOUSEHOLDS IN AN ENDOWMENT ECONOMY WITH HABIT FORMATION.

3.1 DEMOGRAPHICS AND INSURANCE MARKETS

Overlapping generations models are used to study aggregate implications from intertemporal optimization on the part of households. The most tractable model available in the literature is due to Blanchard (1985) and Yaari (1965). We will start this section following Blanchard (1985) and Blanchard and Fisher (1989) quite closely. In the model time is treated as a continuous variable. This assumption is made for analytical convenience only[6]. The model is based on two fundamental sets of assumptions. The first characterizes demographics. We assume a constant probability of death, independent of age, and denote it by ρ. Agents thus have the same life expectancy, regardless of their age. Due to this assumption the model is often referred to as a model of perpetual youth. It is convenient to work under the further assumption that population is constant over time. Thus, we assume that, each period, a new (large) cohort of people are born and that the size of such cohort is constant over time (and has always been so). If we as a simplifying assumption, normalize the size of each cohort at birth to be ρ, it follows that the size of total population is normalized to one (which is convenient). In general, working out aggregate implications from intertemporal optimization on the part of households, is very difficult in overlapping generations models. The reason is that households of different ages will differ not only in their accumulated wealth but also (given differences in life expectancy) in their propensity to consume out of wealth. Perpetual youth simplifies matters by doing away with the second complication, thus allowing for relatively simple aggregation across households. Moreover, the explicit allowance of births affects patterns of expenditure over time in interesting ways. The second fundamental set of assumptions relates to the availability of a perfectly competitive insurance annuities market. Given the demographic assumptions made there is individual risk concerning the timing of death but no aggregate risk. Therefore, there is room for insurance. Specifically, the idea is that individuals will enter a contract with

[6]Frenkel and Razin (1996) present a discrete time version of the model. Many times when performing numerical simulation exercises a discrete time version of the model is used.

insurance companies stipulating that, in case of death, the insurance company will receive the individual's wealth. In exchange, the insurance company will pay the individual an amount equal to ρ times the amount of wealth in case of survival (competition ensures that such actuarially fair terms will be on offer). We make two further simplifying assumptions. The first is that each period all agents, irrespective of age, receive an endowment, in the form of a constant flow ω^T units of a traded good and ω^N units of a non-traded good. Both goods are non-storable. The traded good may be bought or sold on world markets and an exogenous world price, we, for convenience, we set to unity. Agents may borrow from or lend at the interest rate r which - given our open economy assumption - is exogenously given by the world interest rate plus an exogenous risk premium. In addition, as a further simplifying assumption, we postulate that preferences can be represented by a time-separable lograthmitic utility function with θ denoting the intertemporal rate of time preference.

Let us start by writing down our demographic assumptions explicitly. Given the assumed constant mortality rate ρ the probability of dying at age t equals:

$f(t) = \rho \exp(-\rho t)$.

Given that $f(t)$ is the probability density function corresponding to dying at time t it is a standard matter to show that life expectancy is simply ρ^{-1}, which may be interpreted as the relevant economic horizon of economic agents. Given our assumption of mortality rates independent of age it is the case that this horizon is the same for all agents alive. For each individual household the timing of death is uncertain but we also assume that each age cohort has a sufficiently large number of individuals so that the law of large numbers exactly applies. In other words, in each period an eaxct ρ proportion of the population dies. It is this absence of aggregate risk that makes it possible to assume a perfectly competitive annuities market, offering actuarially fair terms to households.

In the remainder of this paper, unless otherwise stated, we will use small caps to denote variables pertaining to individuals, while capital letters will be used for aggregate variables.

3.2 A REPRESENTATIVE HOUSEHOLD INTERTEMPORAL PROBLEM.

We are now ready to write down the problem of a representative household of cohort t. As in all standard applications of theory, households will maximize their expected utility. In our setting, with two goods, one traded good and one non-traded good, logarithmic preferences and habit formation, the household will maximize the present, expected discounted value of utility, which may be written as:

$$\max_{c_t^T, c_t^N} E_t \left[\int_t^\infty \log\left(c_s - \mu H_s\right) \exp\left(-\theta(s-t)\right) ds \right]$$

with c_s being given by a Cobb-Douglas composite of traded and non-traded goods:

$$c_s = \kappa(c_s^T)^\alpha(c_s^N)^{1-\alpha}$$

where t and s denote time, $c_t^T$and c_t^N denote respectively the consumption of traded and non-traded goods, H_s denotes the habit variable, and θ is the subjective discount factor while κ is a parameter. Thus the instantaneous utility of the consumer depends not just on the level of consumption but rather on the level of relative to some reference level, a "normal " or habit consumption level (H_s). Since, in the model, the only uncertainty pertains to the timing of death the objective of the household may be written more simply, using our demographic assumptions to calculate expected values, as:

$$\max_{c_t^T, c_t^N} \int_t^\infty \log\left(c_s - \mu H_s\right) \exp\left(-(\theta+\rho)(s-t)\right) ds.$$

Each household faces a budget constraint, which we may write in the form of accumulation of assets or, if negative, debts. Specifically:

$$\frac{df_t}{dt} = (r+\rho)f_t + p_t^T(\omega^T - c_t^T) + p_t^N(\omega^N - c_t^N).$$

The budget equation is intuitive. The individual household accumulates assets in accordance with the private rate of return on past assets, given by the sum of the interest rate with the compensation for the probability of death *plus* the sum of the value of endowment *minus* consumption expenditure. As we have indicated above we will assume external habit formation so that the individual household takes the sequence of $[H_s]_{s=t}^{s=\infty}$ as given. In particular, in line with, for example, Carroll, Overland, and Weil (2000), we assume that the stock of habit - which is common across consumers - evolves according to:

$$\dot{H}_t = \gamma(C_t - H_t)$$

where $C_t$is aggregate consumption. This equation implies that the reference habit level depends on a geometrically weighted average of current and past consumption in the economy, a form of "keeping up with the Joneses ".

In what follows we will let p^T be the numeraire and set it to 1. Letting $z_t = p_t^T c_t^T + p_t^N c_t^N$ we can express the intratemporal first order conditions for $c^T$and c^N as:

$$c_t^T = \alpha z_t$$

and

$$p_t^N c_t^N = (1-\alpha) z_t$$

These expressions enable us to define a dual price index for aggregate consumption, p_t[7]. The budget constraint then becomes:

$$\frac{df_t}{dt} = (r+\rho)f_t + \omega^T + p_t^N \omega^N - p_t c_t.$$

Applying the maximum principle to solve the consumer's optimisation problem, we obtain the following dynamic first order conditions:

$$\frac{1}{c_t - \mu H_t} = \lambda_t p_t \tag{1}$$

$$\dot{\lambda}_t = \lambda_t(\theta - r) \tag{2}$$

where λ_t is the costate variable.

Letting $x_t = p_t c_t - \mu p_t H_t$ - a variable which may be described as nominal effective consumption - these first-order conditions together with the budget constraint and the transversality condition lead, after tedious but straightforward manipulation, to the individual household's consumption function:

$$x_t = (\theta+\rho)\left(f_t + \frac{\omega^T}{r+\rho} + \int_0^\infty y_s \exp\left(-(\theta+\rho)(s-t)\right) ds\right),$$

Given our assumptions the equation can be simply aggregated to give rise to an aggregate expenditure function (where capital letters refer to variables agregated across all households):

$$X_t = (\theta+\rho)\left(F_t + \frac{\omega^T}{r+\rho} + \int_0^\infty Y_s \exp\left(-(\theta+\rho)(s-t)\right) ds\right).$$

The corresponding equation of motion for the aggregate foreign asset position is:

$$\frac{dF_t}{dt} = rF_t + \omega^T + Y_t - X_t$$

3.3 THE DYNAMICS OF THE AGGREGATE ECONOMY.

Aggregating the relations obtained in the previous subsection, and substantially working on the resulting equations, it is possible to obtain a system of equations describing the aggregate dynamics of the economy. The system consists of three differential equations and two additional definitional equations:

$$\dot{X}_t = (r-\theta)X_t - \rho(\theta+\rho)F_t \tag{3}$$

[7] The formula for the dual price index is $p_t = \left(\frac{1}{\kappa}\right)\left(\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\right) p_t^{\alpha} p_n^{(1-\alpha)}$

$$\dot{F}_t = rF_t + \omega^T - \alpha X_t - \alpha\mu p_t H_t \tag{4}$$

$$\dot{H}_t = \gamma(C_t - H_t) \tag{5}$$

$$X_t = p_t C_t - \mu p_t H_t. \tag{6}$$

$$p_t = \phi^{\frac{1}{\alpha}} C_t^{\frac{1-\alpha}{\alpha}} \tag{7}$$

In deriving (4) we have made use of the market clearing condition for non-traded goods, $\omega^N = p_t^N c_t = (1-\alpha)(X_t + \mu p_t H_t)$. (6) is a definition of aggregate nominal effective consumption. Equation (7) expresses the price index as a function of aggregate real consumption with ϕ being a a constant term which depends on ω^N, α and κ.[8]

Equations (3) to (7) may be reduced to a non-linear three dimensional system of differential equations in $[Z_t, F_t, H_t]$ where Z_t is nominal consumption expenditure (i.e. $Z_t = p_t C_t$):

$$\dot{Z}_t = \frac{(\mu\gamma + r - \theta)\, Z_t - \mu\phi(\gamma + (r-\theta)) Z_t^b H_t - \rho(\theta+\rho) F_t}{\left(1 - b\mu\phi Z_t^{b-1} H_t\right)} \tag{8}$$

$$\dot{F}_t = rF_t + \omega^T - \alpha Z_t \tag{9}$$

$$\dot{H} = \gamma\left(\frac{1}{\phi} Z_t^{1-b} - H_t\right) \tag{10}$$

where b=1-α.

As far as we are aware, no analytical solution is available for this system so it will need to be solved numerically. Before doing so, however, it is useful to gain intuition about the functioning of the model by examining a number of simpler special cases. For these cases closed form analytical solutions can be derived, allowing for a detailed interpretation of the results. This we do in the next sections, starting with the simplest case: the single good case without habit formation.

3.4 SINGLE GOOD WITHOUT HABITS.

In this case case we have $\mu = \gamma = \omega^N = 0$ and $\alpha = 1$. Under these assumptions, the system of differential equations (8), (9), (10) presented at the end of the previous sub-section can be reduced to :

$$\dot{C}_t = (r-\theta)C_t - \rho(\theta+\rho)F_t \tag{11}$$

and

$$\dot{F}_t = rF_t + \omega - C_t \tag{12}$$

[8] This equation is derived from the formula for the dual price index, the first-order condition for non-traded goods consumption and the the market clearing condition for non-traded goods.Specifically, $\phi = \alpha^{-\alpha}\left(\frac{1}{\kappa}\right)\left(\omega^N\right)^{\alpha-1}$

This is the standard model discussed, at length, in Blanchard (1985) and Blanchard and Fisher (1989). (11) and (12) constitute two linear differential equations determining the dynamics of consumption and the net foreign asset position of the economy. The system is linear in C and F and r $< (\theta+\rho)$ is necessary in order for it to be saddle path stable[9]. The dynamics of the system may be analyzed by means of a standard phase diagram (Figure 1). In order to do so it is important to depict the locus, in the space (F_t, C_t), where $\dot{F}_t = 0$ and also the locus where $\dot{C}_t = 0$. It is possible to write those respectively as:

$$\dot{F}_t = 0 : C_t = \omega + rF_t \tag{13}$$

$$\dot{C}_t = 0 : C_t = \frac{\rho(\rho+\theta)}{r-\theta} F_t \tag{14}$$

We can solve (13) and (14) for the steady state values of the net foreign asset position and consumption as:

$$F = \frac{(r-\theta)}{(\theta+\rho-r)(r+\rho)}\omega \tag{15}$$

$$C = \frac{\rho(\theta+\rho)}{(\theta+\rho-r)(r+\rho)}\omega \tag{16}$$

Note that a positive level of consumption implies $r < \theta + \rho$, which is exactly the condition for saddle path stability. In order to determine the impact of a permanent change in the interest rate on these two variables, we differentiate (15) and (16) with respect to r to obtain:

$$\frac{dF}{dr} = \frac{(r-\theta)^2 + (\rho+\theta)\rho}{[(\theta+\rho-r)(r+\rho)]^2}\omega > 0 \tag{17}$$

$$\frac{dC}{dr} = \frac{\rho(\rho+\theta)(2r-\theta)}{[(\theta+\rho-r)(r+\rho)]^2}\omega \tag{18}$$

Equation (17) shows that a fall in the interest rate will always lead to a decline in the steady state value of the net foreign asset position. The derivative in (18) will be positive provided $2r > \theta$ which will likely be the case in practice. Otherwise the implied rate of time preference would be such that the (negative) net foreign asset position would be enormous Thus, in all relevant cases, a lower interest rate will lead to a decrease in steady state consumption.

We can illustrate the adjustment process in Figure 1. We start assuming that $r = \theta$ so that the initial value of the net foreign asset position is zero (as follows from (15)). The inital steady state position is labelled 1. in the figure. In the initial steady state it is clear that consumption is constant over time and equals the endowment (as may be verified from, for example, (16)).

When interest rates decline the locus, correponding to an unchanging net foreign asset

[9]The system has two corresponding eigenvalues $\rho > 0$ and r-$\rho-\theta$.In order for it to be saddle path stable the second eigenvalue must be negative r- $\rho - \theta < 0$.

position becomes more horizontal, turning around the point $C = \omega$. At the same time, the locus corresponding to unchanging consumption becomes visible in the Figure as a negative sloped line in the left hand side quadrant. Given the starting point it is clear that, in steady state, the net foreign asset position becomes negative (the country resorts to foreign debt) and consumption falls. However, on impact the net foreign asset position is pre-determined while consumption is a jump variable. In our case consumption jumps immediately to the level:$C = \frac{\rho+\theta}{\rho+r}\omega$(see Figure 1, point 2). From such starting point consumption will monotonically decline and net foreign assets as well until, eventually, the economy approaches the new steady state equilibrium (point 3 on Figure 1).

To repeat the economy converges to a new steady state in which debt is higher than initially while consumption is lower. In short, what we see, in this simple example, is a form of consumption boom, as consumption significantly increases initially. Subsquently, it gradually declines until it reaches a steady state level lower than its initial level. It is important to note that the dynamics of adjustment is very slow. Specifically, the speed of adjustment is ruled by $r - \rho - \theta$ which is a very small number. For example if we were to consider a 1 percentage point drop in the cost of international financing (starting from $r = \theta$) and a life expectancy of fifty years, the half life of adjustment to the steady state would last for over twenty years. In our simple endowment economy, the pattern stems entirely from the dynamics of consumption and debt accumulation and by construction does not involve capital accumulation or capital adjustment costs.

It is warranted to stress that such a very simple model is apt to reproduce qualitatively many of the stylized facts described in secton 2. Expenditure does increase initially, savings decline and a persistent current account emerges leading to a significant decline in the net foreign asset position. Therefore, the single good endowment model without habit formation already captures the essence of the intertemporal adjustment mechanism. The main limitations of the story have to do with the sudden initial jump in consumption and the extremely large predicted changes in the steady state net foreign asset position. Such enormous external debts have never been observed historically. In the next section we will see how habit formation helps along both these dimensions. In addition, since the model relates by construction to a single good economy, it has nothing to say about the real exchange rate, an important feature of the adjustment in the data, as wasshown in Section 2.

3.5 SINGLE GOOD WITH HABITS.

Now, for the purposes of exposition, let us introduce external habit formation into the economy while retaining the single-good assumption. This is achieved by setting $0 < \mu < 1$ and $\gamma > 0$ while keeping $\alpha = 1$ and $\omega^N = 0$. The introduction of external habit formation changes significantly the dynamics of the economy. Specifically the system of two equations (12) and (11) in the previous sub-section is replaced by the following system of three

equations:

$$\dot{C}_t = (r + \mu\gamma - \theta)C_t + \mu(\theta - r - \gamma)H_t - \rho(\rho + \theta)F_t \tag{19}$$

$$\dot{F}_t = rF_t + \omega - C_t \tag{20}$$

$$\dot{H}_t = \gamma(C_t - H_t) \tag{21}$$

It is important to note that the equation describing the evolution of the net foreign asset position (20) is exactly as before (12). The main difference is given by the equation of motion for consumption. The implication is going to be a more gradual build up of consumption, in response to the reduction in interest rates. More surprisingly, the introduction of habit formation leads to a reduction in the magnitude of the steady state impacts on consumption and the net foreign asset position. The intuition is discussed below. Both of these features of the model with habits help to better mimic the stylized facts reported in Section 2.

As before, in order to solve for steady state values we impose that $\dot{X}_t = 0$ for all state variables (C, F, H). It is clear from (21) that, in steady state consumption will be exactly equal to the stock of habits. Thus, we don't need to worry about determining the steady state value of the stock of habits. Solving the system for C and F leads to:

$$C = \frac{\rho(\rho + \theta)}{\Delta}\omega \tag{22}$$

where $\Delta = -(1 - \mu)r^2 + (1 - \mu)r\theta + \rho(\rho + \theta)$. Similarly the expression for F may be written as:

$$F = \frac{(1 - \mu)(r - \theta)}{\Delta}\omega \tag{23}$$

From (22) and (23) it may be verified that the expressions reduce, as expected, to the case without habit formation for the case $\mu = 0$. From these equations it is straigthforward to derive the effect of a change in interest rates on steady state values. Specifically:

$$\frac{dC}{dr} = \frac{(1 - \mu)\,(2r - \theta)\,\rho\,(\rho + \theta)}{\Delta^2}\omega \tag{24}$$

$$\frac{dF}{dr} = \frac{(1 - \mu)(r - \theta)^2 + (\rho + \theta)\rho}{\Delta^2}\omega > 0 \tag{25}$$

Given that we have $0 < \mu < 1$ the first derivative in (24) will be positive provided that $2r > \theta$,the same condition as we had in the previous section. The fact that the term (1-μ) appears in the numerator of both equations makes it plausible that the introduction of external habit formation will contribute to mitigate the impact of interest rate changes on steady state values. This conjecture is confirmed by a more careful examination of the

relevant expressions. The intuition for this result is as follows. In the model, the Euler equation of individual consumption implies that, when $\theta \neq r$, the consumption profile of an individual household with either be increasing over time (if $\theta < r$) or decreasing over time ((if $\theta > r$). The consumer will therefore choose a path for consumption with this profile which satisfies his intertemporal budget constraint. In steady state consumption per capita is constant and, thus, $\dot{H} = 0$. When habits are introduced, the utility of the consumer depends on the difference between consumption and the level of habit, which in the steady state will be given by $\mu C > 0$, where C is aggregate consumption. This implies that, compared to the case without habits, they will prefer a flatter profile of consumption (i.e. the will not want to their consumption to deviate "too much" from average consumption in the economy). The flatter profile of consumption implies that households will acquire less debt over their lifetimes and thus, aggregating over cohorts, aggregate steady state net foreign debt will be lower. This is why steady state effects are reduced by habit formation. Of course, it is immediately intuitive that external habit formation will moderate the initial adjustment and spread it over time. The intuition here is even simpler since habits are a pre-determined variable. Hence, external habit formation allows us to satisfactorily answer two of the reservations we have raised at the end of section 4.

3.6 TRADED AND NONTRADED GOODS WITHOUT HABITS.

We now consider a variant of the model with traded and non-traded goods but still disregarding habit formation. This corresponds to setting $\omega^N > 0$, $0 < \alpha < 1$ with $\mu = \gamma = 0$. Letting Z_t denote aggregate nominal consumption expenditure ($p_t C_t$), (8) collapses to:

$$\dot{Z}_t = (r - \theta) Z_t - \rho(\theta + \rho) F_t \tag{26}$$

The foreign asset accumulation (9) equation simplifies to:

$$\dot{F}_t = r F_t + \omega^T - \alpha Z_t \tag{27}$$

The equations (27) and (26) are analogous to the equations (12) and (11) that we obtained in the sub-section 3.4. for the single good case. The only difference is that consumption expenditure is measured in nominal terms and the coefficient α in the equation of motion of the net foreign asset position. This difference is immediately intuitive once we recognize that the relevant influence for the accumulation of foreign assets is given by the economy's net supply of tradable goods in world markets.

As before it is convenient to compute the steady state values of F_t and Z_t. In order to do so we follow exacly the steps we used in the earlier section and find the relation between F_t and Z_t that is compatible with unchanging values for each of the variables:

$$\dot{F}_t = 0 : Z_t = \frac{\omega^T}{\alpha} + \frac{r}{\alpha} F_t \tag{28}$$

$$\dot{Z}_t = 0 : Z_t = \frac{\rho(\rho+\theta)}{r-\theta} F_t \tag{29}$$

Equations (28) and (29) are analogous to the corresponding expressions in the previous section (with the qualification that α appears on the denominator of (28). Solving the system for the steady state values of the variables F* and Z* allows us to write:

$$F^* = \frac{r-\theta}{(\theta+\rho-r)(r+\rho\alpha)-(1-\alpha)r\rho}\omega^T \tag{30}$$

$$Z^* = \frac{\rho(\theta+\rho)}{(\theta+\rho-r)(r+\rho\alpha)-(1-\alpha)r\rho}\omega^T \tag{31}$$

It may be verified that equations (30) and equation (31) reduce to the corresponding expressions in section 3.4 in case $\alpha = 1$ (i.e. when all goods are traded). We may now calculate the derivative of F* and Z* with respect to the interest rate, in order to obtain the response of the steady state values of the net foreign asset position and consumption to a change in interest rates. The result may be written as:

$$\frac{dF^*}{dr} = \frac{(r-\theta)^2+\alpha\rho(\rho+\theta)}{[(\theta+\rho-r)(r+\rho\alpha)-(1-\alpha)r\rho]^2}\omega^T \tag{32}$$

$$\frac{dZ^*}{dr} = \frac{\rho(\rho+\theta)(2r-\theta)}{[(\theta+\rho-r)(r+\rho\alpha)-(1-\alpha)r\rho]^2}\omega^T \tag{33}$$

where the derivatives are positive in the same conditions as before. Now for a given interest rate the dynamic system is linear and the conditions for saddle path stability are easy to idendify The saddle path will be given by a straight line whose slope is determined by the value of the normalized eigenvector (see Appendix 2, and Figure 2). Given these preliminary remarks we can now continue and discuss the adjustment of the economy when faced with a reduction in interest rates.

We can illustrate the adjustment process in Figure 2. We start, as before, assuming that $r = \theta$ so that the initial value of the net foreign asset position is zero. The inital steady state position is labelled 1. in figure 2. In the initial steady state it is clear that consumption is constant over time at the level $Z = \frac{\omega^T}{\alpha}$. As the interest rate declines, the locus corresponding to unchanging consumption becomes visible in Figure 2 as a negative sloped line in the left hand side quadrant. Given the starting point it is clear that, in stedy state, the net foreign asset position becomes negative (the country resorts to foreign debt) and consumption expenditure falls. However, on impact the net foreign asset position is pre-determined while expenditure is a jump variable. In our case consumption jumps up immediately (see Figure 2, point 2)[10]. From this starting point nominal consumption

[10]The fact that expenditure jumps upwards follows from the fact that the slope of the saddle path is

and net foreign assets will decline monotonically and the economy will approach the steady state equilibrium (point 3 on Figure 2). The adjustment is entirely analogous to the process described for the case of the simple endowment economy.

However, the version with non-traded goods allows us to discuss the relative price implications from the adjustment process. In the general case, the relative price of non-traded goods is determined by the supply and demand for these goods (see, for example, Neary (1988)), for a compact exposition). In our case, since by assumption the supply of non-traded goods is fixed, then the price of non-traded goods is determined in a relatively straightforward manner by aggregate demand. To show this we note that the intratemporal first order condition for non-traded goods implies that:

$$p_t^N C_t^N = (1-\alpha) Z_t$$

while market clearing in the non-traded goods market requires:

$$C_t^N = \omega^N$$

These two equations imply that:

$$p_t^N = \frac{(1-\alpha) Z_t}{\omega^N} \tag{34}$$

Since endowments are assumed constant over time (34) shows that there is a very simple linear relation between aggregate expenditure and the relative price of non-traded goods. Hence, in our simple set-up, the real exchange rate jumps up on impact and subsequently depreciates very gradually, in line with the slowdown in domestic expenditure (see Figure 2). The new steady state value of the real exchange rate is lower (more depreciated) than in the initial steady-state. This reflects the operation of a 'transfer effect' whereby the higher steady state level of foreign debt depresses domestic expenditure, leading to a lower non-traded goods price.

4 NUMERICAL SIMULATIONS FOR A TWO GOODS ECONOMY.

We now turn to the version of the model outlined at the start of Section 3, equations (8) to (10), which includes both traded and non-traded goods as well as habit formation. We first analyse steady state effects, which are relatively straightforward. However, as regards dynamics, as far as we are aware, analytical solutions to this non-linear equation system are not available so we examine the model's properties by use of numerical simulations. By

greater than the slope of the line connecting the points of unchanging net foreign asset position given by (28). (see Appendix).

way of comparison and in order to show the effects of habit formation, we compare results with the results obtained in the previous section (for a model with two goods but without habit formation). In the simulations reported below, the time period is annual.

Setting $\dot{X}$, $\dot{F}$ and $\dot{H}$ to zero, we can show that the steady state is given by

$$Z^* = \frac{\rho(\rho+\theta)}{\alpha\rho(\theta+\rho) - r(r-\theta)(1-\mu)}\omega^T$$

$$F^* = \frac{(r-\theta)(1-\mu)}{\alpha\rho(\theta+\rho) - r(r-\theta)(1-\mu)}\omega^T$$

where, as before, $Z_t = p_t C_t$. We see immediately, that when habit formation is suppressed, (i.e. $\mu = 0$) we obtain exactly the same results as in the previous section. Differentiating these expressions with respect to r, we obtain:

$$\frac{dZ^*}{dr} = (2r-\theta)\frac{\rho(\theta+\rho)}{[\alpha\rho(\rho+\theta) - r(r-\theta)(1-\mu)]^2}\omega^T$$

$$\frac{dF^*}{dr} = \frac{(1-\mu)((r-\theta)(r(1-\mu)-\theta) + \alpha\rho(\theta+\rho)}{[\alpha\rho(\rho+\theta) - r(r-\theta)(1-\mu)]^2}\omega^T$$

The first expression is positive for $2r > \theta$ while the second is positive for $r \leq \theta$ (which is the case we will be concerned with here). Comparing the expression for $\frac{dZ^*}{dr}$ with the corresponding result for the case without habit, we see immediately that for $r < \theta$ it is smaller, i.e. the long-run response of consumer expenditure is less under habit formation. As regards foreign assets, consider the behaviour of the ratio $\frac{F^*}{Z^*}$. Clearly, the response of this ratio to changes in r is less under habit formation. Thus, to summarize, we obtain the same qualitative long-run effects from the decline in interest rates with and without habit formation. Following a fall in interest rates, expenditure falls while foreign debt rises in the long-run. However, the magnitudes of the long-run effects are smaller under habit formation.

Turning to the dynamics, as in any numerical simulation exercise it is important to examine the sensitivity of the results for different parameter values. For this purpose, we linearise the system around the post-shock steady state and examine the dynamics of the linearised system over a wide grid of plausible parameter values[11].

The main results of this sensitivity analysis are as follows.

First, in all cases, the saddle-path property was satisfied. Thus for every case, one of the roots had a positive real part while the remaining roots had negative real parts.

Second, in around 9% of cases, the roots took the form of complex conjugates. Complex roots are typically associated with values of $\mu > 0.8$.

[11] A total of just over 20,000 different cases were considered. The range of parameters considered was: $r \in [0.03, 0.07]$,$\rho \in [0.01, 0.05]$,$\alpha \in [0.1, 0.9]$,$\mu \in [0.1, 0.9]$,$\gamma \in [0.1, 1.0]$.

Third, in all cases, both real consumption (C) and nominal consumption (Z) jumped upwards on impact. The magnitude of the initial jump however varied greatly depending on the parameter values. In general the size of the jump in consumption is increasing in α but decreasing in μ, γ, r, ρ.

Fourth, the speed of adjustment to the steady state varied significantly across parameter sets. In particular, higher values of ρ - i.e. reduced life expectancy - are associated more rapid adjustment in line with what we found in Section 3.4. Increases in the values of all of the other parameters also led to faster adjustment, though less strongly than in the case of ρ.

With this background, we now present simulations of the model. The shock we consider is a permanent reduction in the interest rate r from 5% to 4%. To implement the shock, we choose a set of parameters which we believe to be plausible. We begin with the intertemporal rate of time preference θ, which we set to 5 per cent. This specific choice is motivated by our desire to match data on net foreign assets. From the steady state results given earlier, we know that a rate of time preference equal to the interest rate implies a level of net foreign assets of zero. This is close to what we observe in the data for the converging countries prior to their entry into the monetary union. We assume a life expectancy of 50 years, corresponding to a ρ of 2 per cent, a standard value in the literature. We calibrate the parameter α to reproduce the expenditure share of traded goods at 30% (so that $\alpha = 0.3$) which is in line with input-output evidence for the countries concerned. The price of traded goods is given in world markets and is chosen as the numeraire. The size of the endowment of traded goods is set to unity. The endowment of non-traded goods is set at a level such that the post-shock steady-state price level is unity. In the case with habit formation there are two extra parameters to calibrate: μ and γ. Following the relevant literature we assume that $\mu = 0.7$ and $\gamma = 1$[12]. We also consider the case in which there is no habit formation in which case we set $\mu = 0.0$ and $\gamma = 0.0$.

Starting from this initial position we consider a disturbance to the cost of financing of the economy in the World's financial markets. Specifically we consider a reduction of one full percentage point in the cost of external financing from 5 per cent to 4 per cent. As argued before such reduction may be interpreted as reflecting a decline in an idiosyncratic risk premium in an uncovered interest rate parity condition or, more broadly, as reflecting a process of financial integration and financial liberalization, leading to easier access to financing for domestic economic agents.

The simulations presented are based on the system (8) to (10) above. Once these equations are solved, obtaining the equilibrium relative price of nontraded goods is straightforward as is the computation of the dual price index, p_t . The system of differential equations is solved by using the MATLAB BVP4C routine, which employs collocation methods to

[12]The value of μ is consistent with the values chosen by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003). In both discrete time models, the authors set $H_t = C_{t-1}$. Applied to our continuous time case, this corresponds to setting $\gamma = 1$.

solve two-point boundary problems for ordinary differential equations. The initial conditions for the two predetermined variables (F and H) correspond to the initial (preshock) steady state while the value of nominal consumption (Z) at the terminal point is given by the postshock steady state value.

The results are presented on Figure 3a (which shows the results for the first 10 years) and Figure 3b (which shows the results for the first 100 years). The results are presented as deviations from baseline except for the current account and the net foreign asset position which are presented as percentages of GDP. In both versions of the model - with and without habit formation - there is an increase in expenditure after the fall in the cost of external financing. This increased expenditure is financed through external debt, implying a current account deficit. The boom in expenditure leads - in the face of unchanged supply of non-traded goods - to an increase in the relative price of non-traded goods or, in other words, to an appreciation of the real exchange rate.

The difference between the two models in the short to medium term is clearly evident in Figure 3a. As expected the response is more muted with habit formation and the initial increase in expenditure is spread over time, in accordance with an inverted U-shaped curve. The response of the current account is smaller, and, thus, the accumulation of foreign debt slower. Since the effects on prices (and the real exchange rate) are basically proportional to aggregate expenditure the same remarks apply to prices. With habit formation the adjustment process looks much more realistic in the sense that the initial adjustment does not involve a large immediate jump in expenditure.

Moreover, there are also important differences concerning steady state results, as we would expect from the analytical results given earlier. Nevertheless, both versions of the model have in common that the adjustment process is very slow, as can be seen in Figure 3b. The reason is intuitive. Given the fall in the cost of external financing, interest rates are now lower than the households' rate of time preference. Thus, individual consumption expenditures will be declining over time. So, in order for aggregate consumption expenditures to be constant (in steady state) the level of consumption of newborns has to be sufficiently above the average consumption in the economy. This can only be the case after a sufficient accumulation of foreign debt as taken place, and this takes place only slowly.

For the case without habit formation, steady state results are very large. Specifically, for our parameter values, the model implies a steady state value of foreign debt (a negative net foreign asset position) equal to 550 per cent of GDP. Aggregate real consumption is 18 per cent lower in the new steady state. The relative price of non-traded goods falls by 50 per cent and, correspondingly, the true consumption price index falls by 38 per cent.

With habit formation all the qualitative results are similar. However, habit formation makes all magnitudes much smaller. For example, the level of foreign debt with habit formation is reduced by almost a factor of three to 200 per cent of GDP. We think that these latter results are much more plausible.

5 CONCLUSION.

In this paper, we have argued that, for a number of countries, it is useful to look at the process of adjustment to euro area participation as involving a significant fall in the costs associated with international financing. The simplest way to think about such disturbance is as a fall in a country specific risk premium leading to a drop in the relevant interest rate. It is, however, best to interpret it in broader terms, reflecting better conditions of access to financing for domestic economic agents, in part related to a process of financial integration and financial liberalization which accompany entry into the monetary union. In the paper, we argue that the fall in the cost of financing was a common impulse affecting countries like Greece, Spain, Ireland, Italy and Portugal. In order to establish the relevant stylized facts we compare these converging countries with the remainder of the countries now constituting the euro area. In the paper, we document that the process of adjustment in these economies shared a number of important common features. These include: an increase in aggregate expenditure and a fall in savings, the emergence of a significant current account deficit, an appreciation of the real exchange rate, and, finally, the accumulation of households' debt and of a negative net foreign asset position. Surprisingly no significant changes in the pattern of GDP growth differentials can be discerned in the data.

Hence we argue that it is possible to explain the observed patterns using a simple intertemporal general equilibrium model for an endowment economy. Specifically, we consider an economy constituted by Blanchard-Yaari households. We show that all the stylized facts can be qualitatively obtained in a simple endowment economy model with traded and non-traded goods. However, in versions with standard time-separable preferences, aggregate expenditure jumps instantaneously and the impact on the steady state net foreign asset position seems to be too large. We show that introducing external habit formation enables us to solve both problems. Therefore, we conclude that this version of the model goes a long way in explaining some important features of adjustment to the euro.

Appendix 1. Data used in Section 2

The main data source for the results presented in Section 2 is the EU Commission's AMECO database. The codes for the series employed are as follows: nominal short-term interest rates (ISN), hnominal long-term interest rates (ILN), household savings rate (ASGH), nominal housing investment (UIGDW), current account balance (UBCA), HICP (ZCPIH), whole economy net lending (UBLA), general government net lending (UBLG), household net lending (UBLH), nominal GDP (UVGD) andreal GDP (OVGD). Non housing investment is calculated as the difference between total investment (UIGT) and housing investment.

The exceptions to the use of AMECO are as follows. Credit to households data came from the OECD Financial Balance Sheet databank (except in the case of Ireland where the the source was various issues of the Central Bank of Ireland Quarterly Bulletin). Data on net foreign assets are taken from Lane and Milesi-Ferreti (2006). Data on the Harmonised Index of Consumer Prices (HICP) were taken from Eurostat while the data on relative consumer prices come from Eurostat's Comparative Consumer Price Level dataset and relate to price levels for final consumption by private households. For Portugal and Ireland, the data on net lending shown in Chart 13 are from the Banco de Portugal and the authors own calculations respectively.

Ratios to GDP were calculated by dividing the respective series (as described above) by nominal GDP. The variables for core countries were calculated by taking cross-country arithmetic means of the respective variables for Belgium, Germany, France, Austria and the Netherlands. Converging country variables were similarly calculated using data for Ireland, Italy, Portugal and Spain.

Appendix 2. Further results on the model with two goods and without habits.

In this Appendix, we establish that consumption expenditures (Z_t) jump upwards following a fall in the interest rate. As is clear from Figure 2 in the main text, this requires that the slope of the saddle path is more vertical than the slope of the $\dot{F} = 0$ locus (28). Thus, our task in the Appendix is to show that saddle path stability implies the initial jump in consumption expenditures.

Let us start by noticing that the pair of equations (26) and (27) can be expressed in matrix form as:

$$\begin{pmatrix} \dot{Z}_t \\ \dot{F}_t \end{pmatrix} = \begin{pmatrix} (r-\theta) & -\rho(\theta+\rho) \\ -\alpha & r \end{pmatrix} \begin{pmatrix} Z_t \\ F_t \end{pmatrix} + \begin{pmatrix} 0 \\ \omega^T \end{pmatrix}$$

Let

$$A = \begin{pmatrix} (r-\theta) & -\rho(\theta+\rho) \\ -\alpha & r \end{pmatrix}$$

Saddle path stability requires that the the determinant of A be negative, i.e.

$$r(r-\theta) - \rho\alpha(\theta+\rho) < 0$$

Let $\lambda_1 < 0$ denote the stable root of A. Then the solution of the model is given by:

$$\begin{pmatrix} Z_t - Z^* \\ F_t - F^* \end{pmatrix} = b \begin{pmatrix} v_{11} \\ 1 \end{pmatrix} e^{\lambda_1 t}$$

where b is a constant determined to match the initial value of the predetermined variable, $(v_{11}, 1)'$ is the normalised eigenvector associated with $\lambda_1$and a star on the variable denotes the steady state value. The equation for the saddle path is therefore:

$$Z_t - Z^* = v_{11}\left(F_t - F^*\right)$$

and its slope is therefore v_{11}. Now $v_{11}$is determined by the solution of:

$$\begin{pmatrix} (r-\theta-\lambda_1) & -\rho(\theta+\rho) \\ -\alpha & r-\lambda_1 \end{pmatrix} \begin{pmatrix} v_{11} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

This implies that

$$v_{11} = \frac{r-\lambda_1}{\alpha} > 0$$

Let η denote the slope of the $\dot{F} = 0$ locus (28). Clearly

$$\eta = \frac{r}{\alpha}$$

We have then:

$$v_{11} - \eta = \frac{-\lambda_1}{\alpha} > 0$$

because, as noted above, $\lambda_1 < 0$.So the slope of the saddle path is greater than the slope of the $\dot{F} = 0$ locus, as required.

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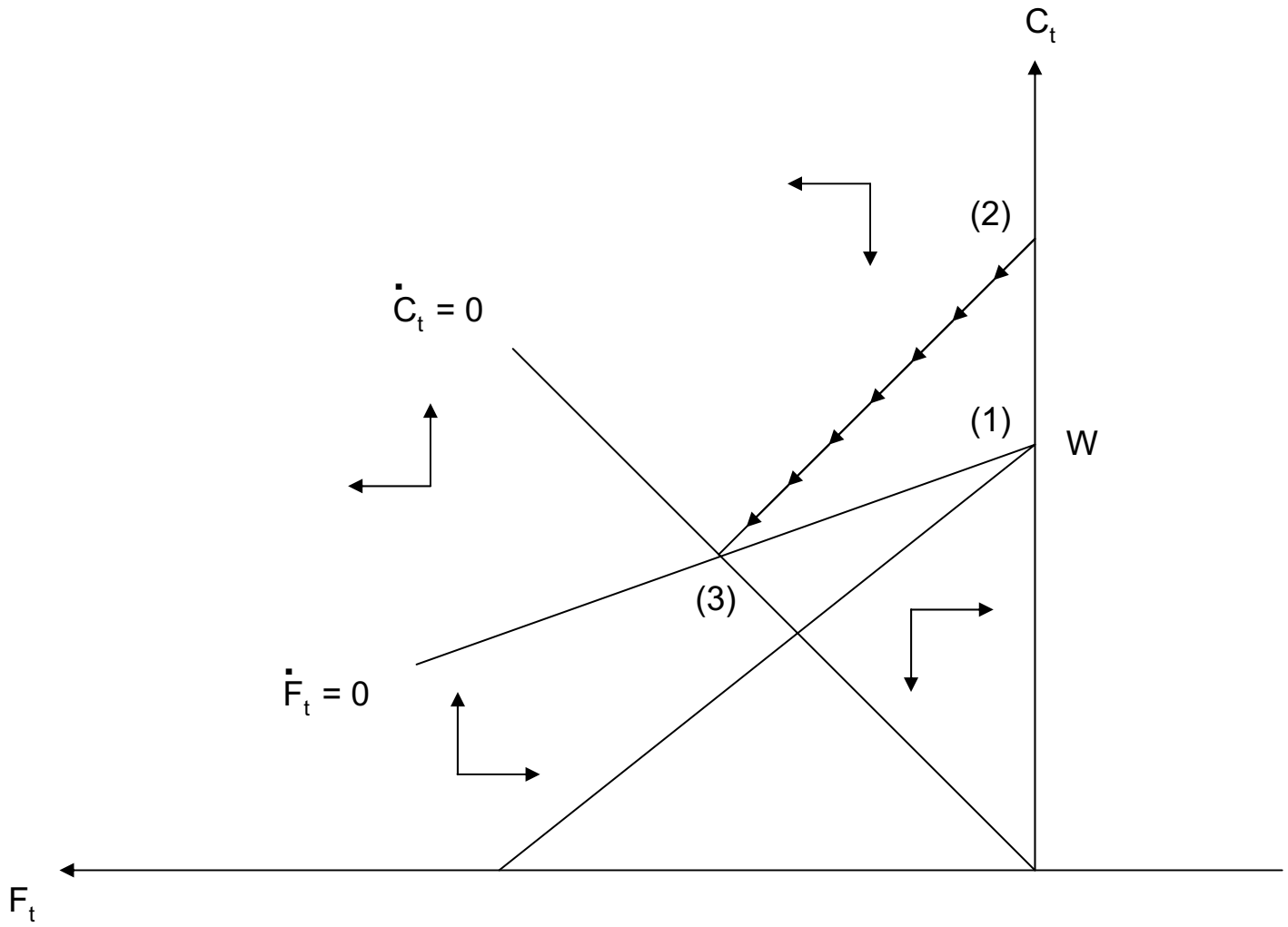


Fig.1: Dynamics of consumption and net foreign assets.

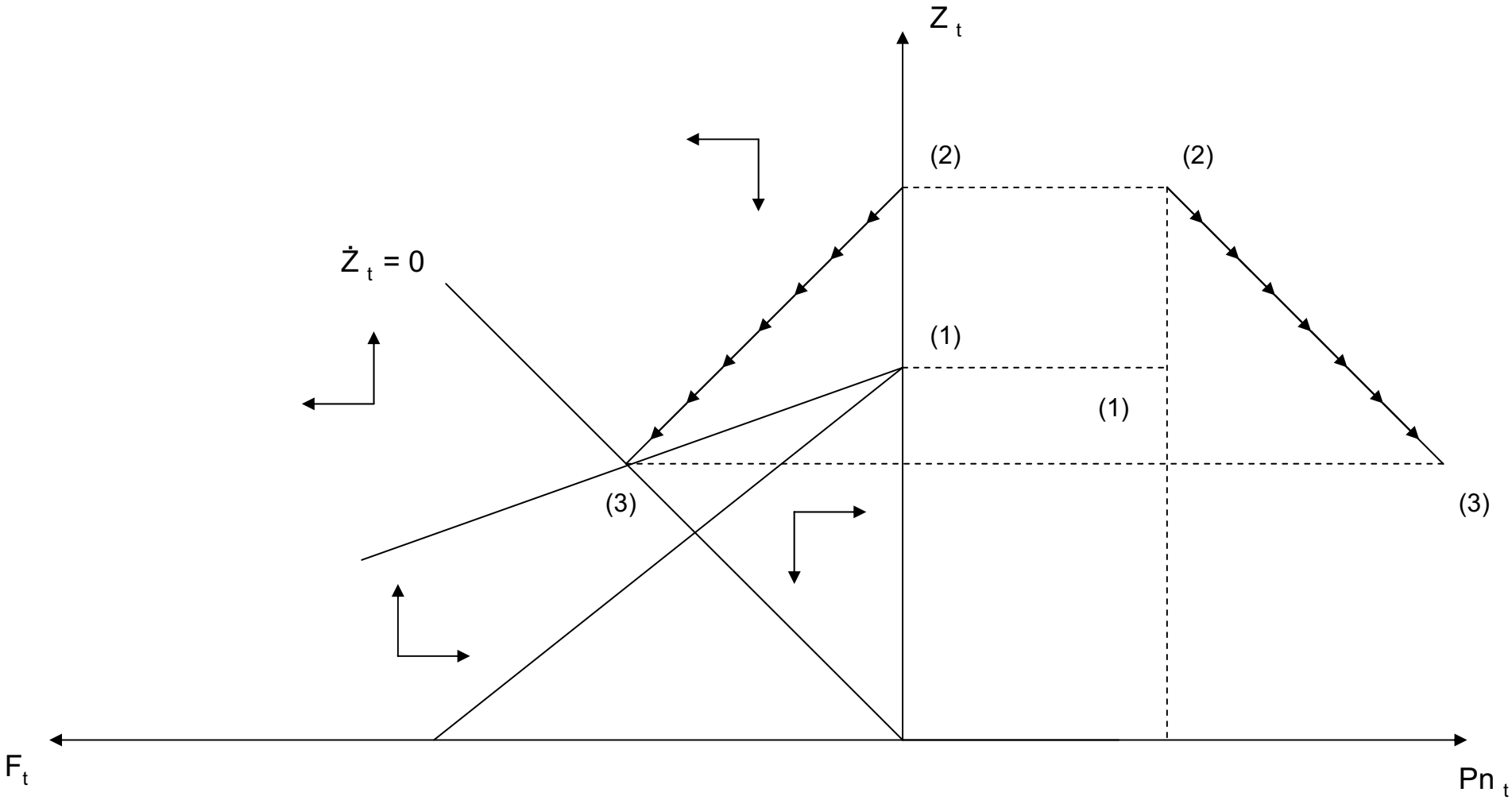


Fig.2 : Dynamics of consumption, net foreign assets and the real exchange rate.

Figure 3a: Simulated response to a fall in interest rates (first 10 years)

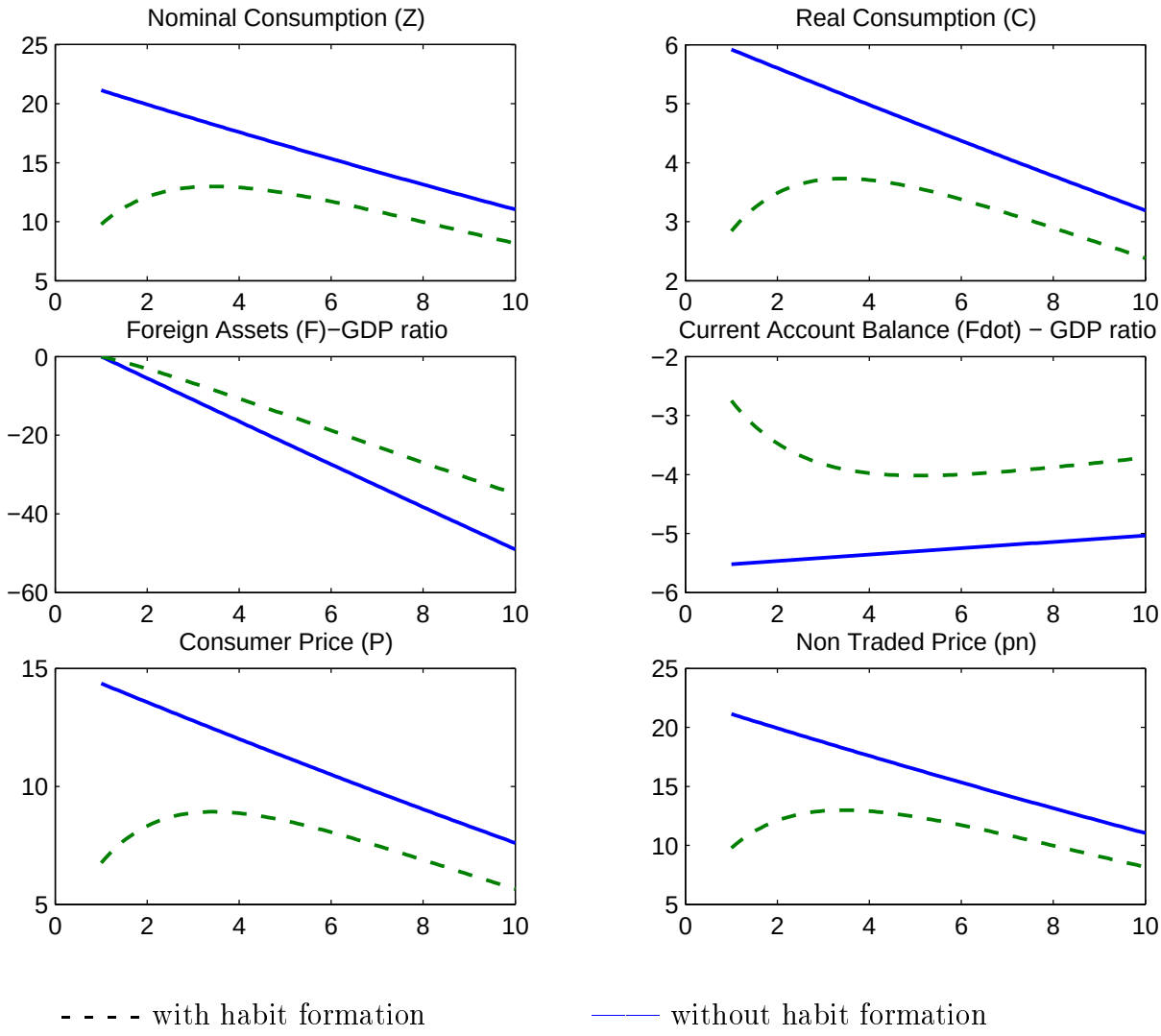
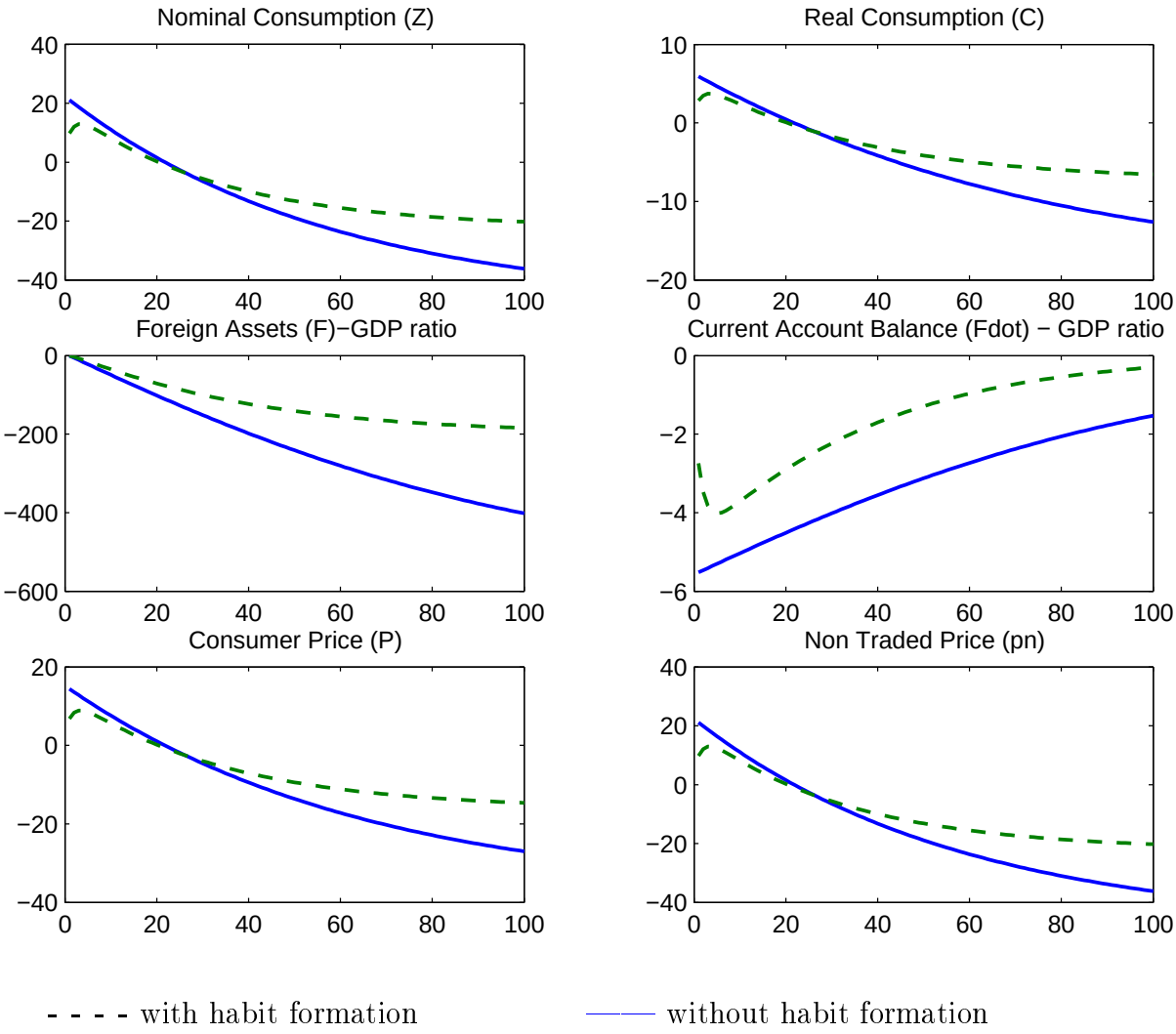
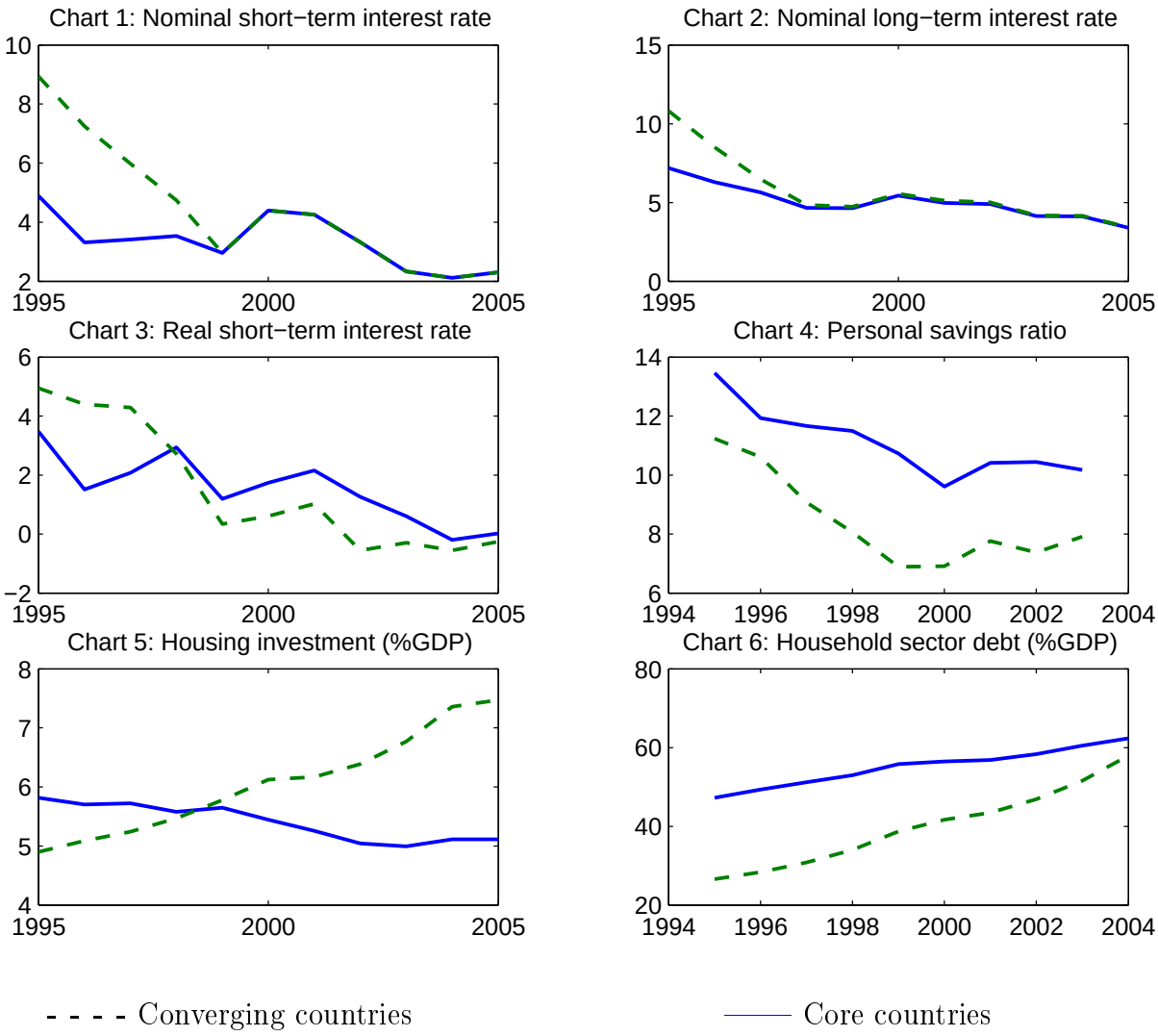


Figure 3b: Simulated response to a fall in interest rates (100 years)

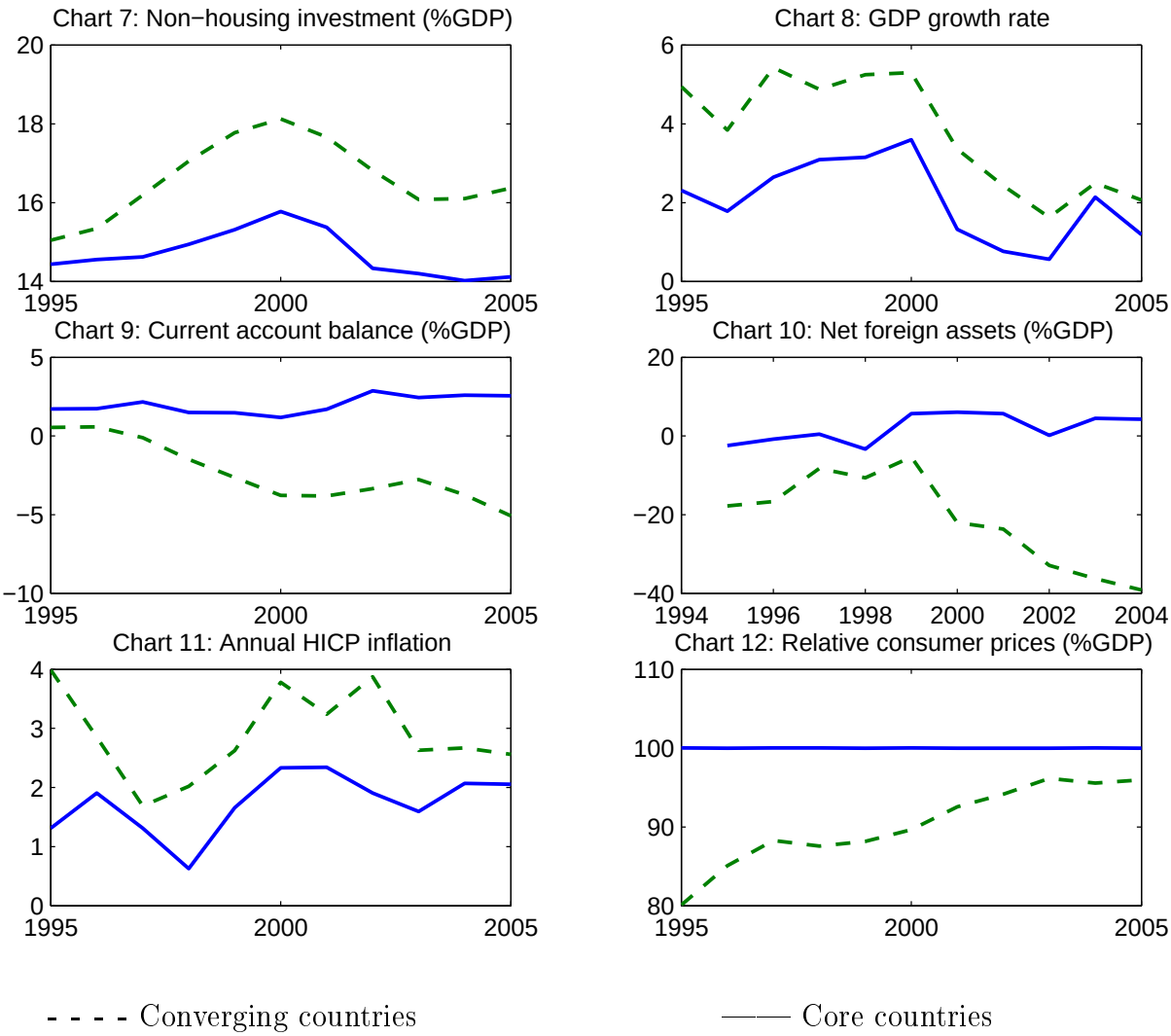




- - - - Converging countries

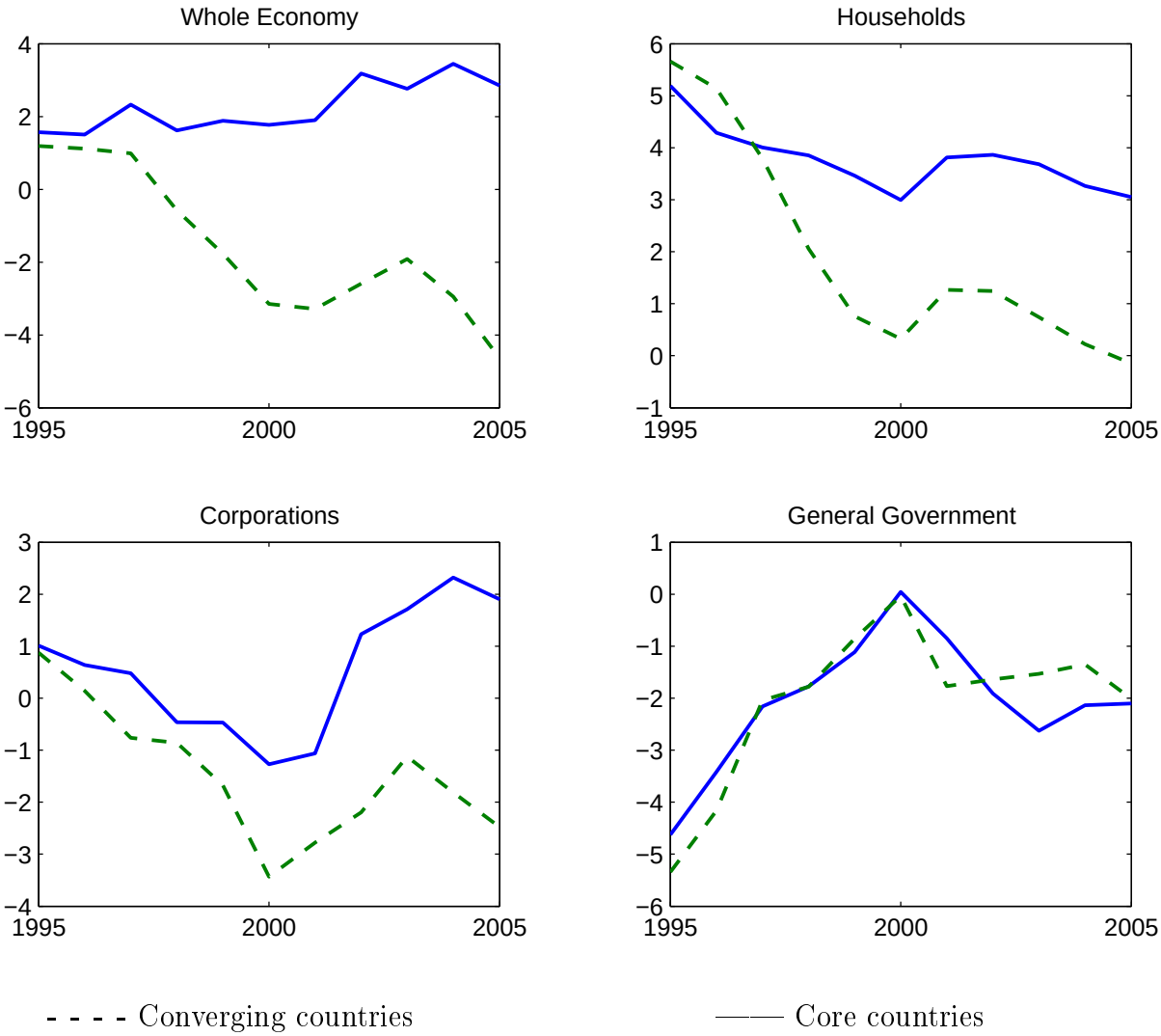
—— Core countries

The charts display averages for each variable and country group. "Core countries" includes Austria, Belgium, France, Germany and the Netherlands. "Converging countries" includes Ireland, Italy, Portugal and Spain.



The charts display averages for each variable and country group. "Core countries" includes Austria, Belgium, France, Germany and the Netherlands. "Converging countries" includes Ireland, Italy, Portugal and Spain.

Chart 13: Net Lending by Sector (percent of GDP)



The charts display averages for each variable and country group. "Core countries" includes Austria, Belgium, France, Germany and the Netherlands. "Converging countries" includes Ireland, Italy, Portugal and Spain.