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# **Purchasing Power Parity and Heterogeneous Mean Reversion**

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# **Purchasing Power Parity and Heterogeneous Mean Reversion**

## **Abstract**

This paper analyzes the properties of multivariate tests of purchasing power parity (PPP) that fail to take heterogeneity in the speed of mean reversion across real exchange rates into account. We compare the performance of homogeneous and heterogeneous unit root testing methodologies. The recent literature has successfully contested several severe restrictions on the structure of the model, but the assumption of homogeneous mean reversion is still widely used and its consequences are virtually unexplored. Using Monte Carlo simulation, we uncover important adverse properties of the methodology that relies on homogeneous estimation and testing. More specifically, power functions are low and assume irregular shapes. Furthermore, homogeneous estimates of the mean reversion parameters exhibit potentially large biases. This can have a dramatic impact on inferences made on the validity of the PPP hypothesis. Our findings highlight the importance of allowing for heterogeneous estimation when testing for a unit root in panels of real exchange rates.

## **Keywords**

Purchasing power parity, real exchange rates, panel models, unit root tests, heterogeneity

## **JEL subject codes**

F31, F33, G15

## 1. Introduction

Since the early 1990s, researchers interested in testing the hypothesis of purchasing power parity (PPP) have turned to multivariate testing procedures in order to increase the statistical power.<sup>1</sup> Initial applications of multivariate analysis to real exchange rates imposed severe restrictions on the structure of the model. Two of these restrictions have been successfully challenged in the recent literature. First, O'Connell (1998) questions the common assumption in the PPP literature that the real exchange rates are cross-sectionally independent.<sup>2</sup> He demonstrates that spurious rejections of the unit root null can occur when cross-sectional dependence is neglected. In response to his critique, nearly all subsequent research relaxes this restriction and thus takes cross-sectional dependence into account. Second, Papell and Theodoridis (2001) and Wu and Wu (2001) criticize the prevalent restriction that the serial correlation properties of all real exchange rates in the panel are the same. Both papers show that assuming a restrictive homogeneous serial correlation structure weakens the evidence against the unit root null. In line with these findings, recent panel studies on PPP abandon this second restriction as well.

Yet, the use of a third important restriction on the structure of multivariate models of real exchange rates is still widespread in the academic literature on PPP. The vast majority of recent empirical studies assume a common mean reversion coefficient across all real exchange rates.<sup>3</sup> From an economic perspective, the justification for the assumption that PPP holds equally well for all country pairs is weak. The speed of mean reversion of a real exchange rate between two countries should depend on, for example, their relative proximity, their mutual trade regulations, and the openness of their economies. The econometric consequences of imposing homogeneous mean reversion for the properties of multivariate PPP tests have not been thoroughly investigated to date. Exploratory econometric research on the properties of panel data models (notably Robertson and Symons (1992) and Pesaran and Smith (1995)) suggests that the homogeneity assumption in dynamic panel models may have serious consequences. Pooling heterogeneous panel data can lead to biases in the parameter estimates, as a result of which estimation results are

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<sup>1</sup> Examples of early multivariate PPP studies include Abuaf and Jorion (1990), Frankel and Rose (1996), and Jorion and Sweeney (1996).

<sup>2</sup> Specifically, O'Connell (1998) examines the restriction that the variance-covariance matrix of the residuals in the panel model is diagonal.

<sup>3</sup> See e.g. Choi (2004), Fleissig and Strauss (2000), Frankel and Rose (1996), Jorion and Sweeney (1996), Levin, Lin, and Chu (2002), Lopez and Papell (2004), O'Connell (1998), Papell (1997), Papell (2002), and Papell and Theodoridis (2001).

potentially misleading. An important question is to what extent this affects the inferences drawn from multivariate studies on the PPP hypothesis.

This paper analyzes the finite-sample properties of various multivariate unit root tests employed for investigating PPP. In order to assess the consequences of the homogeneity restriction, we compare three different multivariate estimation and testing methodologies. The first methodology involves homogeneous estimation of the mean reversion parameters and a unit root test on the validity of PPP for the full panel of real exchange rates. This methodology, or a variation thereof, is applied by a large number of recent empirical papers on PPP (examples are provided in footnote 3). The second methodology entails estimating the model heterogeneously, but still testing the PPP hypothesis jointly for all series in the panel. This means that while any inferences about PPP still concern the panel as a whole, differences in mean reversion across countries are allowed for. This approach is taken by, among others, Taylor and Sarno (1998), Im, Pesaran, and Shin (2003), and Wu and Wu (2001). In addition, we propose an alternative methodology in which both estimation and testing are performed heterogeneously.<sup>4</sup>

We employ Monte Carlo simulation in order to examine the empirical performance of the three multivariate methodologies. Our Monte Carlo experiments are based on a sample of the real exchange rates between five of the world's largest economies (Canada, the Euro area, Japan, the U.K., and the U.S.) over the period 1978:12-2003:12. We demonstrate that when the assumption of homogeneous mean reversion in the panel of real exchange rates is violated, the methodology that relies upon homogeneous estimation and testing suffers from important adverse properties. First, homogeneous estimates of the mean reversion parameter can exhibit serious biases. Second, large estimation uncertainties arise as a result of the homogeneity restriction. This implies that the statistical power of the homogeneous test against the unit root null is generally limited. Third, the power function is not monotonically increasing when the mean reversion parameters generated under the alternative hypothesis move away from the unit root null. These results indicate that a homogeneous estimation methodology can lead to potentially misleading inferences about the validity of the PPP hypothesis. Our paper offers critical insights into the consequences of imposing parameter restrictions in multivariate tests for PPP by showing that these testing

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<sup>4</sup> In order to correct for the implications of the two restrictions on multivariate models studied by O'Connell (1998) and by Papell and Theodoridis (2001) and Wu and Wu (2001), our implementation of all three methodologies takes account of cross-sectional dependence as well as heterogeneous serial correlation.

methodologies should not only take cross-sectional dependence and heterogeneous serial correlation into account, but also heterogeneous mean reversion.

The remainder of the paper is structured as follows. In section 2 we describe the methodology. Section 3 provides the data description. The results of our Monte Carlo simulations are presented in section 4. Section 5 concludes.

## 2. Methodology

### 2.1 Multivariate tests of PPP

This section discusses the three different multivariate estimation and testing methodologies we examine in this study. For each country (currency)  $i$  ( $i=1,\dots,N$ ) we define the log real exchange rate at time  $t$  ( $t=1,\dots,T$ ) as follows:

$$R_{i,t} = e_{i,t} - e_{0,t} + p_{0,t} - p_{i,t} \quad (1)$$

where  $R_{i,t}$  is the logarithm of the real exchange rate,  $e_{i,t}$  is the logarithm of the nominal exchange rate expressed in units of currency  $i$  per dollar,  $e_{0,t}$  is the logarithm of the nominal exchange rate expressed in units of the numeraire currency  $0$  per dollar,  $p_{0,t}$  is the logarithm of the consumer price index of the country used as numeraire, and  $p_{i,t}$  is the logarithm of the consumer price index in country  $i$ .

The three methodologies analyzed in this paper are all based on the following multivariate regression:

$$R_{i,t} = \alpha_i + \beta_i R_{i,t-1} + \sum_{k=1}^{l_i} \gamma_{i,k} \Delta R_{i,t-k} + u_{i,t} \quad (2)$$

where  $\beta_i$  are the mean reversion parameters,  $\alpha_i$  are the intercept parameters,  $\gamma_{i,k}$  are the coefficients on the lagged real exchange rate returns,  $l_i$  denotes the number of lags needed for currency  $i$ , and  $u_{i,t}$  is a stationary error term.<sup>5</sup> The null-hypothesis of a unit root is expressed by  $H_0: \beta_i = 1$  for all currencies  $i$ . The three methodologies differ in the estimation of the parameters in equation (2) as well as in the way the test of the unit root hypothesis is performed. Note,

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<sup>5</sup> The value of  $l_i$  is determined by the recursive  $t$ -statistic procedure of Campbell and Perron (1991) applied to each individual log real exchange rate. This means that for currency  $i$  we choose the value of  $l_i$  by first setting  $l_i$  to some maximal value,  $l_{max}$ , then estimate equation (3) by OLS and subsequently test whether the last included lag is statistically significant. If so, then  $l_i$  is set to this value, else the model is estimated by setting  $l_i$  to  $l_{max} - 1$ . The procedure is repeated until a significant value of  $l_i$  is found. When no lag is significant then  $l_i$  is set to 0. Following Wu and Wu (2001), we set  $l_{max}$  to 24 and use a 10% significance level.

however, that for all three methodologies we incorporate the suggestions made by O’Connell (1998) to allow for contemporaneous correlations between the error terms  $u_{i,t}$  and by Papell and Theodoridis (2001) and Wu and Wu (2001) to allow the serial correlation parameters to vary across exchange rates.

In the first methodology, estimation of equation (2) and testing the unit root hypothesis are carried out homogeneously. That is, the restriction  $\beta_i = \beta$  is imposed for all  $i$  and the null-hypothesis and alternative hypothesis can thus be expressed as  $H_0: \beta = 1$  and  $H_A: \beta < 1$ , respectively. This implies that the PPP hypothesis is only evaluated for the panel as a whole and not for individual country pairs. A large number of recent empirical papers adopt (a variation of) this methodology (see footnote 3). For instance, O’Connell (1998) applies this methodology in order to study the restriction that exchange rates are cross-sectionally independent. Accounting for cross-sectional dependence, he finds no empirical evidence for PPP in a panel of 64 countries over the period 1973:I-1995:IV. As a second example of homogeneous estimation and testing, Lopez and Papell (2004) investigate convergence towards PPP within the euro zone and between the euro zone and other countries. Using data over the period 1973:I-2001:IV, they present evidence of convergence toward PPP within the euro zone, but not for the real exchange rates of the euro versus other currencies.

The second methodology we investigate involves heterogeneous estimation of the mean reversion parameters  $\beta_i$  in equation (2), but still testing the unit root hypothesis in a homogeneous way. This means that while inferences about PPP concern the entire panel and no statements can be made about individual country pairs, the speed of mean reversion is allowed to differ across countries. The null-hypothesis can be expressed as  $H_0: \beta_i = 1 (\forall i = 1, \dots, N)$ , while there are several possibilities for the alternative, for example,  $H_A: \exists j \in \{1, \dots, N\}: \beta_j < 1$  or  $H_A: \forall i \in \{1, \dots, N\}: \beta_i < 1$ . The interpretation of the first alternative is that at least one of the exchange rates is stationary, the second states that all exchange rates are mean-reverting. This implies that rejection of the unit root null does not necessarily provide information on how many and which real exchange rates in the panel are stationary. An example of a study using this methodology is Taylor and Sarno (1998). They estimate an equivalent multivariate model as in equation (2) and perform a joint unit root test on all  $N$  equations by constructing a standard Wald test statistic, which they refer to as the Multivariate Augmented Dickey-Fuller (MADF) test. The alternative hypothesis for this test is  $H_A: \exists j \in \{1, \dots, N\}: \beta_j < 1$ . Critical values for this test are obtained by

Monte Carlo simulation. The MADF test rejects the unit root null in a panel containing the UK, France, Germany, Japan, and the U.K. (with the U.S. as numeraire country) over the period 1973:I-1996:II. Taylor and Sarno (1998) suggest an alternative testing methodology, based on the Johansen likelihood ratio (JLR) test for cointegration, which tests the unit root null hypothesis versus the alternative that  $H_A: \forall i \in \{1, \dots, N\}: \beta_i < 1$ . The JLR test also rejects the null hypothesis of non-stationarity for their panel of exchange rates. Other papers that use a methodology in which estimation is performed heterogeneously, but the unit root null hypothesis is defined for all real exchange rates in the panel simultaneously, are Im, Pesaran, and Shin (2003) and Wu and Wu (2001). Im, Pesaran, and Shin (2003) develop a unit root test with the same null hypothesis as described above, but with alternative hypothesis  $H_A: \beta_i < 1, i = 1, \dots, N_I, \beta_i = 1, i = N_I+1, \dots, N$ . Hence, a rejection of the null-hypothesis in this model does not necessarily mean that all exchange rates in the panel are mean-reverting. Wu and Wu (2001) apply the tests developed in the paper of Maddala and Wu (1999) and in an earlier version of the paper of Im, Pesaran, and Shin (2003). Both tests are based on univariate ADF regressions and have  $\forall i \in \{1, \dots, N\}: \beta_i < 1$  as alternative hypothesis.<sup>6</sup> Wu and Wu (2001) take heterogeneous serial correlation and cross-sectional dependence into account and document substantial evidence for PPP in a panel of 20 industrial countries over the period 1973:II-1997:IV.

The third methodology entails both heterogeneous estimation and heterogeneous testing of equation (2). The unit root null-hypothesis and alternative hypothesis for every real exchange rate  $i$  ( $i = 1, \dots, N$ ) are  $H_0: \beta_i = 1$  and  $H_A: \beta_i < 1$ , respectively. In this framework, the PPP tenet is evaluated for each individual real exchange rate in the panel. Flôres, Jorion, Preumont, and Szafarz (1999) apply such a methodology and report evidence in favor of long-run PPP for the G10 currencies versus the U.S. dollar in the period 1973:01-1994:12. Koedijk, Tims, and van Dijk (2004) enhance this approach by allowing for heterogeneous serial correlation. They investigate a panel of ten countries within the euro area over the period 1973:02-2003:03 as well as for the real exchange rates of the euro versus other major currencies over the period 1978:12-2003:03. The empirical evidence shows that the mean reversion properties differ importantly across real exchange rates. Three out of the nine real exchange rates within the euro area are mean reverting and only one out of the eight real exchange rates versus the euro is stationary.

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<sup>6</sup> Note that this alternative hypothesis is different from the one used in the published version of the Im, Pesaran, and Shin paper.



## 2.2 Estimation procedure

All three methodologies rely upon Seemingly Unrelated Regression (SUR) estimation of equation (2). After determination of the number of lags  $l_i$  (for each currency  $i$ ), the estimation procedure is as follows. First, for each currency  $i$ , we apply OLS to equation (2). Second, the resulting covariance matrix of the error terms is used as the weighting matrix in a Feasible Generalized Least Squares (FGLS) procedure to estimate the full panel.<sup>7,8</sup> When applicable, we impose the homogeneity restriction in the second step. For the methodology in which the model is estimated and tested homogeneously, we compute the usual ADF test statistic  $\tau = (\beta-1)/s(\beta)$  to evaluate the unit root null-hypothesis. In order to test the unit root hypothesis homogeneously when estimation is carried out heterogeneously, we employ the MADF test as described in Taylor and Sarno (1998). For the methodology with heterogeneous estimation and testing, inferences about the stationarity of the individual real exchange rates are based on the individual ADF  $\tau$ -statistics  $\tau_i = (\beta_i-1)/s(\beta_i)$ .

We use Monte Carlo simulations to derive critical values for the test statistics of the various multivariate tests. In the first step, given the estimated parameters of the model  $\alpha_i$ ,  $\beta_i$ ,  $\gamma_{i,k}$ , and  $l_i$  we compute residuals and the corresponding covariance matrix. Second, we generate  $N$  error terms  $u_{i,t}$  ( $T$  times) from a multivariate normal distribution with mean zero and this covariance matrix. Third, given the estimated parameters  $\gamma_{i,k}$  and  $l_i$ , assuming  $\alpha_i = 0$ , and imposing the null-hypothesis  $\beta_i = 1$ , we compute simulated exchange rate series using equation (2) on the basis of the simulated error terms  $u_{i,t}$  from step 2. Fourth, given the value of  $l_i$ , we estimate the parameters in equation (2) with the simulated exchange rate series and compute the test statistic. We replicate step 2 to step 4 1,000 times and derive critical values for the test statistic from its sample distribution. Alternatively, empirical  $p$ -values can be calculated as the fraction of times the observed test statistic using the actual empirical data series is exceeded in the replications.

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<sup>7</sup> The estimation procedure can be extended by iterating between estimating parameters, constructing the corresponding residuals, and using the covariance matrix of these residuals in the FGLS step. However, for computational reasons (following Taylor and Sarno (1998)), we limit this iterative process to one iteration. Unreported results show that this does not have an important effect on the resulting estimates.

<sup>8</sup> In a standard SUR model the degrees of freedom needed for the calculation of the covariance matrix of the error terms equals  $T-2$ . We have to make a correction for the inclusion of lagged changes in the real exchange rate in the model. To that effect, we set the degrees of freedom to  $T-2-\text{entier}[(l_i+l_j+1)/2]$ , where  $\text{entier}[x]$  rounds  $x$  down to the nearest integer value.

### *2.3 Monte Carlo simulations*

Our purpose is to investigate the finite-sample properties of the three methodologies, including potential estimation biases and the statistical power of the unit root test. Our approach is inspired by Taylor and Sarno (1998) and is based on Monte Carlo experiments. We simulate real exchange rate series for several data generating processes (DGP's) using equation (2) as described above but instead of setting  $\beta_i = 1$  for all exchange rates to construct the distribution of the ADF test statistic under the unit root null-hypothesis (see step 3 of the previous paragraph), we assign other values to the mean reversion parameters. We follow Taylor and Sarno (1998) and use the following set of values for  $\beta_i$ :  $\{0.99, 0.975, 0.95, 0.925, 0.9\}$ . These rates of mean reversion correspond to half-lives of PPP deviations amounting to, respectively, 69, 27, 14, 9, and 7 months. The power functions of the tests are subsequently constructed by comparing the simulated values of the test statistic to the simulated critical value under the null-hypothesis. To gain further insight into the power properties of the multivariate tests, we monitor the estimate of the mean reversion parameter and the estimate of the uncertainty of this parameter. In particular, we assess the bias of the mean reversion parameter estimate that can arise because of the homogeneity restriction (see e.g. Robertson and Symons (1992) and Pesaran and Smith (1995)) as well as the small sample (see e.g. Murray and Papell (2002)).

In order to avoid confusion in our description of the properties of the three different methodologies in subsequent sections, we introduce the following notational shorthand. At the risk of being labeled the Santa Claus paper, we refer to the first methodology as the HoHo methodology (homogeneous estimation and testing), to the second as the HeHo methodology (heterogeneous estimation and homogeneous testing), and to the final as the HeHe methodology (heterogeneous estimation and testing).

### **3. Data**

We collect consumer price index (CPI) and nominal exchange rate data for Canada, the euro area, Japan, the U.K., and the U.S. for the period 1978:12-2003:12. CPI data and period-ending exchange rates against the U.S. dollar are obtained from International Financial Statistics. Because the euro – dollar rate is only available from January 1999, we use the “synthetic” euro

from the ECB.<sup>9</sup> In order to construct the CPI data for the euro area we employ the geometric weighted average method as described in Maeso-Fernandez, Osbat, and Schnatz (2001, p. 11). Ireland is discarded because the CPI is only available as of 1997. The first 25 observations are used to compute the lagged exchange rate changes needed for the ADF tests. This implies that 276 time-series observations are used for the estimation of the model.

Figure 1 shows the development of the log real exchange rates against the U.S. dollar over the period January 1981 through December 2003. As is obvious from the plots, the selection of countries in our sample has not been driven by the aim to maximize the heterogeneity of the mean reversion properties of the real exchange rates in our panel. In particular, the real exchange rates of the euro and the British pound against the U.S. dollar seem to exhibit very similar time-series patterns. Based on the graphs, both series also appear to share common characteristics with the dollar – yen series, while only the behavior of the Canadian dollar versus the U.S. dollar looks markedly different. The similarities across the real exchange rates are to a considerable extent driven by the large swings in the relative value of the dollar in the 1980s as well as in the past five years. Using a different numeraire currency could potentially lead to more pronounced heterogeneity in the mean reverting behavior of the real exchange rates in the panel. Further heterogeneity would probably be introduced by adding countries that are less well integrated with the world economy, such as emerging markets in Asia, Latin-America, and Eastern Europe.<sup>10</sup> Thus, we bias the results against finding important adverse consequences resulting from the homogeneity restriction in multivariate models.

Table 1 presents summary statistics for the real exchange rates as well as results of univariate ADF unit root tests. As indicated by Figure 1, the correlations between the real exchange rates of the euro, pound, and yen against the dollar are substantial. This underlines the importance of accounting for cross-sectional dependence in panel tests of PPP. Moreover, the bottom panel of Table 1 shows that the serial correlation properties are not identical across the real exchange rate series. The optimal lag length in the univariate unit root tests, determined with the procedure of Campbell and Perron (1991), varies from 13 for the euro – dollar series to 22 for the yen – dollar rate. Hence, it is important to allow for heterogeneous serial correlation.

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<sup>9</sup> This synthetic euro series is the “ECB reference exchange rate, US Dollar/Euro, 2:15 pm (C.E.T.), against ECU up to December 1998,” which is available at the website of the European Central Bank ([www.ecb.int](http://www.ecb.int)).

<sup>10</sup> Note that heterogeneity can also arise within closely integrated economic regions. Koedijk, Tims, and van Dijk (2004) present evidence that real exchange rates within the euro area exhibit important differences in mean-reverting behavior.

Notwithstanding the similarities among three of the four real exchange rate series in Figure 1, the estimates of the  $\alpha_i$  and  $\beta_i$  parameters in equation (2) show considerable differences across the countries examined. The estimated half-life of PPP deviations ranges from about 1 year (13 months) for the U.K. to almost 3.5 years (41 months) for Canada. The unit root hypothesis can be rejected for the U.K. at the 5% level and for the Euro area at the 10% level.

#### 4. Empirical analysis

This section investigates the finite-sample properties of the three multivariate methodologies discussed in section 2. We do not intend to offer an exhaustive treatment of the properties of unit root tests in heterogeneous panels. Rather, the goal is to illustrate how homogeneous estimation and testing of a panel can affect the inferences, in particular when examining the PPP hypothesis in a panel consisting of real exchange rates.

For all three methodologies we estimate the parameters of model (2) and test the PPP hypothesis by comparing the (multivariate) unit root test statistics to their corresponding simulated critical values. Table 2 displays the results of estimating and testing PPP with the HoHo methodology, imposing homogeneous mean reversion. The half-life is estimated at roughly 2.5 years and the unit root hypothesis is rejected ( $p$ -value = 0.023). This suggests that the PPP tenet is a reasonable description of the panel as a whole. Note that the  $\tau$ -test and the Wald test are equivalent in case the unit root hypothesis is tested homogeneously. Table 3 presents the results for the HeHo and HeHe methodologies. Both methodologies require heterogeneous estimation of the  $\alpha_i$  and  $\beta_i$  parameters. Again, the estimated half-lives of PPP deviations differ importantly across the real exchange rates, varying from less than 1.5 years to almost 3.5 years. In the HoHo methodology, this heterogeneity is obscured by the common mean reversion parameter estimate of 29 months. Table 3 shows much less clear-cut evidence in favor of PPP. The Wald (MADF) test rejects the unit root null only at the 10% level for the panel as a whole (HeHo methodology) and the ADF  $\tau$ -statistic (HeHe methodology) is only significant for the euro – dollar rate (5% level).

Inferences regarding the validity of the PPP hypothesis in our panel are highly dependent on the chosen methodology. While the HoHo methodology yields strong rejection of the unit root hypothesis, the HeHe methodology indicates that only one of the four real exchange rates is stationary. Adopting the HeHo methodology leads to an intermediate result, a rejection of the unit

root at the 10% level. A thorough understanding of the consequences of methodology choice requires an analysis of the finite-sample properties of the unit root test across the three methodologies. Therefore, we perform a Monte Carlo study on the empirical performance of the three methodologies. Naturally, when comparing the performance of alternative methodologies to evaluate unit roots in panels, the most interesting experiments concern DGP's that contain both stationary series and series with unit roots. Hence, we follow Taylor and Sarno (1998) and analyze the properties of the methodologies when, respectively, one, two, and three of the simulated series have a root less than unity. Subsequently, we study a variety of DGP's that contain only stationary exchange rates.

#### *4.1 Monte Carlo evidence on panels with both stationary and non-stationary series*

In this section we examine DGP's in which one or more of the real exchange rates in the panel contain a unit root under the alternative hypothesis. First, we consider DGP's with three non-stationary series. The mean reversion parameter ( $\beta_i$ ) of the remaining real exchange rate is set equal to values from the set  $\{0.99, 0.975, 0.95, 0.925, 0.9\}$ . We then apply the HoHo, HeHo, and HeHe methodologies to estimate the parameters in equation (2) and compute the test statistics. Comparing these statistics with the critical values yields a power function for each of the methodologies. We repeat this procedure for all four exchange rates. Figure 2 displays the power functions of the HoHo and HeHo tests for the stationarity of the panel as a whole as well as the HeHe tests evaluating the stationarity of the four exchange rate series individually. Chart titles indicate which of the simulated real exchange rate series is stationary. The first thing that leaps to the eye is the remarkable behavior of the power function of the unit root test of the HoHo methodology. The power of this methodology to reject the unit root null is very low and often even lower than the size of the test (5%). Furthermore, if the  $\beta$  of the mean reverting exchange rate decreases from 0.99 to 0.9, the power is not monotonically increasing. Especially for Canada, the power decreases when the  $\beta$  moves further away from the unit root null. This is remarkable, as the power of the test should increase when the panel as a whole becomes more stationary.

The behavior of the four individual unit root tests in the HeHe methodology is essentially as expected. The power to reject the unit root null for the non-stationary series is very low and roughly equal to the 5% probability of a type I error. The power function of the HeHe test for the stationary series clearly increases when  $\beta$  moves further away from unity. The power to reject the

null depends on the particular real exchange rate that is stationary. For example, for a half-life of 14 months ( $\beta = 0.95$ ) the power is 0.83 for the real exchange rate of the euro area, while it is only 0.56 for the exchange rate of Japan. Hence, the influence of the country or currency of study on the power of the test seems to be substantial. Note that although the power for the HeHe test is the highest of all methodologies, even at low half-lives there is a relative high probability of failing to reject the true null-hypothesis.

The power of the HeHo testing methodology increases when  $\beta$  decreases, but is considerable lower than the power of the HeHe methodology. For example, when the real exchange rate of Canada is mean reverting (see top left graph), the HeHe methodology exhibits a power of 0.75 at a half-life of 14 months, while the HeHo methodology has a power of only 0.45. Although this is a natural implication of the fact that the HeHe test is an individual unit root test while the HeHo methodology evaluates the stationarity of the panel as a whole, it also underlines the role the chosen methodology can play in assessments of the PPP hypothesis.

Figure 3 and 4 show the power of the unit root tests for the three different methodologies when the number of exchange rates in the panel that are non-stationary is equal to two and one, respectively. The mean reversion parameters of the remaining stationary series decrease simultaneously from 0.99 to 0.9. We observe that although the power of the HoHo methodology improves slightly with fewer unit roots in the sample, the power function again does not monotonically increase when  $\beta$  is reduced from 0.99 to 0.9. For example, when the real exchange rates of Canada and Japan are stationary, the power of the panel unit root test is twice as high when the  $\beta$ 's of Canada and Japan are equal to 0.975 compared to the case where both  $\beta$ 's equal 0.9. The irregular properties of the HoHo methodology seem to become more pronounced when the number of unit roots in the panel is reduced.<sup>11</sup>

The power functions of the HeHe methodology reveal that the power does not crucially depend on the number of non-stationary series in the panel, but it does still vary considerably across the currencies. Again, the power of this test to reject the null is very close to the size of the test for the non-stationary series. The power of the HeHo methodology increases when the number of unit roots in the panel decreases, because the panel as a whole becomes more

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<sup>11</sup> Note that the degree of heterogeneity in the panels analyzed in Figure 3 (4) is actually relatively limited, as the mean reversion parameters are identical for two (three) out of the four series. The drawbacks of the HoHo methodology could be aggravated by introducing more heterogeneity in the panel and hence we bias the result against finding important differences between the alternative methodologies.

stationary. When comparing the power properties of the HeHe and the HeHo methodologies, we note that the power is similar for DGP's in which two out of the four exchange rates are stationary. When only one of the exchange rates in the panel contains a unit root, the power of the HeHo test is generally higher.

This paper aims at studying the small sample properties of various alternative methodologies for evaluating the PPP hypothesis for a panel of real exchange rates. Our analysis of the power functions demonstrates that the HeHo methodology is generally most powerful when there are three or more stationary series in the panel. The HeHe methodology succeeds relatively well in identifying individual stationary series. The power properties of the HoHo methodology can be described as undesirable, as the power is low and often not monotonically increasing when the mean reversion parameter decreases. In order to gain more understanding of these adverse properties and thus of the consequences of imposing homogeneous mean reversion, we analyze the distribution of estimated  $\beta$  coefficients under the HoHo methodology.

Figures 5, 6, and 7 display the distribution of the (homogeneously) estimated mean reversion parameter  $\beta$  under the HoHo methodology when the panel contains, respectively, three, two, and one unit roots. The mean reversion parameter of the stationary exchange rates in the panel are assumed to decrease simultaneously from 0.99 to 0.5.<sup>12, 13</sup> We added  $\beta = 0.5$  (corresponding to a half-life of merely 1 month) to the set of  $\{0.99, 0.975, 0.95, 0.925, 0.9\}$  in order to highlight the impact of the homogeneity restriction in extreme circumstances. The histograms also presents the values of  $\mu$ , defined as the average of the true mean reversion parameters, and  $b$ , the (homogeneously) estimated mean reversion parameter.

The results are striking. We observe that when the mean reversion parameter of the stationary series in the panel decreases, the distribution of the estimated  $\beta$  of the total panel first gradually moves to the left. This can be expected, as the panel as a whole becomes more stationary. However, when the mean reversion parameter reaches a value of 0.95 or less, there is a clear movement to the right, implying higher estimates of the homogeneous  $\beta$ . As the value of  $b$

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<sup>12</sup> The distributions are based on the same Monte Carlo simulations as for the analysis of the statistical power of the tests, but 10,000 instead of 1,000 simulations are used to obtain a smoother representation of the distributions.

<sup>13</sup> Note that for a given number of (non-)stationary exchange rates in the panel, there are several possibilities to assign the unit root(s) to the series. For example, in Figure 5 we choose the Canadian dollar – U.S. dollar series to be stationary, but we could have chosen any of the three other series. The results for combinations other than displayed in Figures 5 to 7 are similar and available from the authors.

slowly increases, while the mean of the true mean reversion parameters ( $\mu$ ) steadily drops, this implies an increasingly large bias in the homogeneous parameter estimate. Second, marked changes to the shape of the distribution occur. Moving from the top graphs to the bottom, the dispersion and skewness of the distribution initially increase notably, signifying increasing uncertainty about the parameter estimate. However, when the stationary series in the panel become even more mean reverting, the distribution again becomes more dense and less skewed. This “boomerang” effect is more severe when the panel contains fewer series with unit roots. Hence, the more the panel as a whole moves further away from the unit root null-hypothesis, the more the distribution of the estimated  $\beta$  starts to resemble the one under the null-hypothesis again. Remarkably, the homogenous estimation methodology seems to “ignore” mean reversion parameters that deviate considerably from the non-stationary null and the unit roots in the panel dominate the stationary series in the estimation.

The extraordinary character of this feature of the HoHo methodology is best illustrated by the bottom panel of Figure 7. This panel concerns a DGP in which only the euro – dollar rate contains a unit root, and all other three real exchange rates have half-lives of 1 month ( $\beta = 0.5$ ). The mean of the estimated mean reversion parameters (denoted by  $b$ ) under the HoHo methodology is 0.993, implying a bias of no less than 0.368 relative to the average of the true  $\beta$ 's in the panel under the null-hypothesis ( $\mu = 0.625$ ). These findings indicate that imposing a common mean reversion coefficient in heterogeneous panels can lead to testing methodologies with seriously adverse properties, including low and non-monotonic statistical power and severe biases in the parameter estimates.

#### *4.2 Monte Carlo evidence on panels with only stationary series*

In this section we investigate the power functions of the HoHo, HeHo, and HeHe methodologies for DGP's in which all real exchange rates are stationary. As a benchmark, we first investigate the situation in which all four exchange rates exhibit the same mean-reverting behavior. That is, the mean reversion parameters  $\beta_i$  ( $\forall i$ ) are equal to the same value  $\beta \in \{0.99, 0.975, 0.95, 0.925, 0.9\}$ . The results are displayed in Figure 8. Not surprisingly, the HoHo methodology performs very well. Its power is the highest of all methodologies and it monotonically increases with a decrease in the mean reversion parameter. This can be explained by the fact that the homogeneous model is not misspecified for this DGP. The HeHo methodology is only slightly



less powerful than the HoHo methodology, however. For  $\beta$ 's of 0.95 and lower (corresponding with half-lives of 14 months and less), both testing methodologies are able to reject the unit root in virtually all cases. When the half life is around 2.5 years ( $\beta = 0.975$ ), the probability that non-stationarity is rejected still amounts to 92% (HoHo) and 79% (HeHo). The power functions of the HeHe methodology behave similarly to those generated under DGP's with both stationary and non-stationary series. Homogenous estimation and testing thus leads to a power advantage when the real exchange rates indeed have the same mean-reverting behavior.

As panel data sets in which the series have exactly the same mean reversion parameters are unlikely to be encountered in practice, more interesting DGP's involve real exchange rates with different rates of mean reversion.<sup>14</sup> In Figure 9 we examine the power properties of the multivariate methodologies by setting the  $\beta_i$ 's equal to the estimated values of the heterogeneous SUR model (see Table 3) and decrease the mean reversion parameter of one of the series at the time. As the DGP's in each of the four panels of Figure 9 are generated under the alternative hypothesis, it is not surprising that the methodologies that assess the stationarity of the panel as a whole have the highest power to reject the null. The HeHo methodology generally achieves the highest statistical power. The HoHo methodology is also relatively powerful, but exhibits the same properties as discussed above, notably for the situation in which the mean reversion of the pound – dollar rate is varied (bottom right panel). The power functions of the HeHe methodology have regular shapes: increasing for the exchange rate of which the mean reversion parameter decreases and essentially flat for the other series.

Figure 10 displays the power functions of the three methodologies for DGP's with a decreasing value of  $\beta_i$  for one of the exchange rates, while we assign the values {0.99, 0.975, 0.95} to the remaining mean reversion parameters (these three values are assigned to the remaining real exchange rate series in such a way as to maximize the resemblance with the estimated values in Table 3). The plots demonstrate how sensitive the behavior of the HoHo test is to heterogeneity in the panel. The heterogeneity in the DGP's of Figure 10 is only moderately larger than the heterogeneity in the DGP's of Figure 9, but the power of the HoHo methodology

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<sup>14</sup> Even if we restrict ourselves to parameters from the set {0.99, 0.975, 0.95, 0.925, 0.9}, the number of DGP's we can generate is very large. In Figures 9 and 10 we therefore present the power functions of the HoHo, HeHo, and HeHe methodologies for an interesting subset of all possible DGP's. More results can be obtained from the authors.

deteriorates considerably and the non-monotonicity effect becomes stronger. The power functions of the HeHo and HeHe methodologies are basically unaffected.

We take a closer look at the empirical performance of the HoHo model by plotting the distribution of the estimated  $\beta$  coefficients for the DGP's analyzed in Figure 10. To preserve space, Figure 11 only shows the distributions for the case in which the mean reversion of the Canadian dollar – U.S. dollar rate is varied. The distributions for the other exchange rates display a similar pattern. The “boomerang” effect documented in the previous section is evidently not contingent on the presence of unit roots in the panel. Again, the distribution of the estimated common mean reversion coefficient at first shifts to the left and becomes more dispersed. For lower half-lives of the Canadian dollar – U.S. dollar real exchange rate, however, the distribution moves back to the right.<sup>15</sup> Even without a unit root in the panel, the least stationary series appear to dominate stationary series with a root that deviates from unity. Consequently, the HoHo methodology can also lead to serious biases in the estimated mean reversion parameters in panels with only stationary real exchange rates.

In order to illustrate the differences between the finite-sample properties of the HoHo and HeHe methodologies, we also plot the distribution of the estimated  $\beta$ 's under the HeHe methodology. Figure 12 presents histograms for the heterogeneously estimated  $\beta$ 's for the same DGP as in the top-left panel of Figure 10 and as in Figure 11. Again,  $\mu$  denotes the average of the true mean reversion parameters, and  $b$  represents the heterogeneously estimated mean reversion parameter for the specific series under examination. Note that for clarity we use different  $x$ -axes for the  $\beta$  distributions for Canada. As the half-life of the exchange rate of Canada versus the U.S. increases, the distribution of the estimated  $\beta$  for Canada shifts to the left, while the distributions for the other countries virtually stay the same. For Canada, the estimate  $b$  moves in lockstep with the decreasing true mean reversion parameter  $\beta$ . For the euro area, Japan, and the U.K., the true mean reversion parameters are equal to  $\{0.975, 0.99, 0.95\}$  respectively, while – independent of the decrease in Canada's  $\beta$  – the estimated mean reversion parameter  $b$  equals  $\{0.965, 0.98, 0.934\}$ . This indicates that the HeHe methodology leads to a relatively accurate estimate of individual mean reversion parameters in heterogeneous panels, even in small samples.

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<sup>15</sup> Note that the accompanying decrease in dispersion is less pronounced than in Figures 5-7.

### 4.3 Evaluation of the three methodologies

The evidence presented in sections 4.1 and 4.2 suggests that restricting the mean reversion parameter to be homogeneous across different real exchange rates can have detrimental consequences for the validity of the outcomes of multivariate unit root tests. Irregularly shaped power functions and potentially large biases in the parameter estimates arise in panels with and without non-stationary series and even when the degree of heterogeneity in the panel is relatively limited. These findings constitute compelling reasons for using a methodology that allows for heterogeneous *estimation* of the mean reversion parameters

The issue whether researchers interested in the PPP hypothesis should also perform unit root *tests* heterogeneously depends on the purpose of the study. If the hypothesis of interest is primarily whether one or more of the real exchange rates in the panel are stationary, the researcher's main objective is probably to achieve maximum power. The relative power of the HeHo and HeHe methodologies is dependent on the number of unit roots present in the panel. As this number is unknown, it may be valuable to use both methodologies.

An important drawback of homogeneous tests of PPP is that rejecting the unit root hypothesis does not provide any guidance as to how many real exchange rates are stationary, let alone which.<sup>16</sup> Furthermore, from many perspectives the relevant question is not whether a panel of real exchange rates as a whole can be considered to be stationary, but whether PPP holds between individual countries.<sup>17</sup> Research that is directed at this question should use a methodology that evaluates the PPP hypothesis for individual currency pairs in the sample. An important additional advantage of such a testing methodology is that the power is independent of the number of non-stationary series in the panel.

Alternative approaches to evaluate the PPP hypothesis for individual exchange rates include studying long-run time series (see e.g. Edison (1987) and Lothian and Taylor (1996)) and Bayesian analysis of unit roots in real exchange rates (see e.g. Koop (1992)). Bayesian methods treat the unit root and stationarity hypotheses symmetrically and allow for an assessment of the probability of a unit root in the data by evaluating the Bayesian posterior odds ratio.

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<sup>16</sup> This pitfall is also discussed by Taylor and Sarno (1998). They suggest a test statistic that only rejects the null hypothesis if all real exchange rates are stationary, but this test neither reveals how many nor which series are stationary when the null is not rejected.

<sup>17</sup> This is essential for many practically oriented studies of PPP. As an example, asset managers may be interested in the validity of PPP in relation to hedging specific currency risks.

## 5. Conclusions

Froot and Rogoff (1995) show that a researcher interested in testing the PPP hypothesis needs 72 years of stationary (monthly) data to reject the null-hypothesis of a random walk when the real exchange rate has a half-life of three years. Since the early 1990s, many researchers have turned to multivariate unit root tests in order to overcome this power problem. However, inferences drawn from multivariate testing methodologies are adversely affected by a number of restrictions imposed on the multivariate model. Two of these restrictions have been defied by O'Connell (1998), who emphasizes the importance of allowing for cross-sectional dependence, and by Papell and Theodoridis (2001) and Wu and Wu (2001), who show that it is vital to take heterogeneous serial correlation into account. However, a third critical restriction in multivariate tests for PPP is still widely used: numerous studies impose a homogeneous rate of mean reversion across all real exchange rates in the panel. The economic rationale for this restriction is arguable, as the validity of PPP between two countries should depend on various economic, institutional, and possibly even geographic characteristics specific to those countries. Moreover, imposing this restriction may affect the test outcomes in important ways that have not been comprehensively investigated in the literature.

This paper studies the finite-sample properties of three different multivariate estimation and testing methodologies. We examine the consequences of the homogeneity restriction on tests for PPP by means of Monte Carlo simulation. The first methodology we investigate involves homogeneous estimation of the mean reversion parameters and performing a unit root test on the panel as a whole. Estimation is performed heterogeneously but testing is done homogeneously in the second methodology. In the third methodology, the mean reversion parameters are estimated heterogeneously and the null-hypothesis of non-stationarity is tested for each individual exchange rate in the sample individually.

Our findings uncover important adverse properties of the methodology with homogeneous estimation and testing in the presence of heterogeneity in the data generating process. The statistical power of this testing methodology is relatively low and, remarkably, does not rise when the mean reversion parameter is decreased. In addition, we document significant biases in the estimated mean reversion parameters. In particular, when one or more of the mean reversion parameters are decreased while the remaining parameters stay the same, the

homogenous mean reversion parameters estimate increasingly underestimates the average degree of stationarity in the panel.

These properties are observed for both panels with and panels without non-stationary real exchange rates and arise even when the heterogeneity across the real exchange rates in the panel is limited. The homogeneity restriction only leads to higher power when all exchange rates are stationary and mean reversion parameters are very similar. The power functions of the other two methodologies behave in a regular way. Heterogeneous estimation and homogeneous testing generally leads to substantial power, while the third, fully heterogeneous, methodology performs well in detecting stationarity for individual series, especially when the number of unit roots in the panel is high. These findings highlight the importance of taking heterogeneous mean reversion into account in multivariate tests of PPP.

Our study is related to recent work by Imbs, Mumtaz, Ravn, and Rey (2005). They demonstrate that heterogeneity in the dynamics of sectoral price indices may induce significant biases in estimates of mean reversion parameters based on aggregated price indices. Our analysis shows that when aggregate real exchange rates are used for testing the PPP hypothesis, it is important to recognize that mean reversion properties may differ across countries.

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**Table 1**  
**Summary Statistics and Univariate Unit Root Tests**

This table presents summary statistics and the results of the univariate ADF unit root tests of the real exchange rates of Canada, the Euro area, Japan, and the U.K. versus the U.S. over the period 1981:01-2003:12. We estimate the following equation:

$$R_{i,t} = \alpha_i + \beta_i R_{i,t-1} + \sum_{k=1}^{l_i} \gamma_{i,k} \Delta R_{i,t-k} + u_{i,t},$$

where  $R_{i,t}$  is the log of the real exchange rate and the value of  $l_i$  is determined by the recursive  $t$ -statistic procedure of Campbell and Perron (1991). The critical values have been obtained from Dickey and Fuller (1979). \* and \*\* denote the significance at the 10% and 5% level, respectively.

Country	Mean	St. dev.	Skewness	Kurtosis	Correlation coefficient			
					Canada	Euro Area	Japan	
Canada	0.0401	0.1171	0.1368	2.1163				
Euro Area	0.0769	0.1568	0.4770	2.3235	0.3925			
Japan	-0.1078	0.2087	0.4063	2.5411	-0.0106	0.7553		
U.K.	0.2523	0.1235	0.8751	4.4304	0.1438	0.7368	0.5362	
	$\alpha_i$	s.e.	$\beta_i$	s.e.	half-life	$l_i$	$\tau_i$	
Canada	0.0003	0.0009	0.9833	0.0079	41	16	-2.10	
Euro Area	0.0022	0.0021	0.9657	0.0127	20	13	-2.70*	
Japan	-0.0021	0.0023	0.9819	0.0108	38	22	-1.69	
U.K.	0.0135	0.0048	0.9464	0.0178	13	21	-3.01**	
10% critical value								-2.57
5% critical value								-2.88



**Table 2**  
**Multivariate unit root test: Homogeneously estimated mean reversion coefficient**

This table presents the results of the SUR ADF unit root test (under the HoHo methodology) of the real exchange rates of Canada, the Euro area, Japan, and the U.K. versus the U.S. over the period 1981:01-2003:12. We estimate the following system:

$$R_{i,t} = \alpha_i + \beta R_{i,t-1} + \sum_{k=1}^{l_i} \gamma_{i,k} \Delta R_{i,t-k} + u_{i,t},$$

where  $R_{i,t}$  is the log of the real exchange rate and the value of  $l_i$  is taken from the OLS ADF unit root test results. The HoHo methodology imposes the restriction that the mean reversion coefficient does not vary across countries. The empirical  $p$ -values as well as the critical values are obtained using Monte Carlo Simulations. \* and \*\* denote significance at the 10% and 5% level, respectively.

Country	$\alpha_i$	s.e.	$\beta_i$	s.e.	half-life	$l_i$	$\tau_i$ [ $p$ -value]	Wald [ $p$ -value]
<b>Canada</b>	0.0005	0.0009	0.9765	0.0056	29	16	-4.23**	18.34**
<b>Euro area</b>	0.0016	0.0019				13	[0.023]	[0.023]
<b>Japan</b>	-0.0027	0.0021				22		
<b>U.K.</b>	0.0061	0.0023				21		
10% critical value							-3.59	13.33
5% critical value							-3.95	15.99

**Table 3**  
**Multivariate unit root tests: Heterogeneously estimated mean reversion coefficients**

This table presents the results of the SUR ADF unit root test (under the HeHo and HeHe methodologies) of the real exchange rates of Canada, the Euro area, Japan, and the U.K. versus the U.S. over the period 1981:01-2003:12. We estimate the following system:

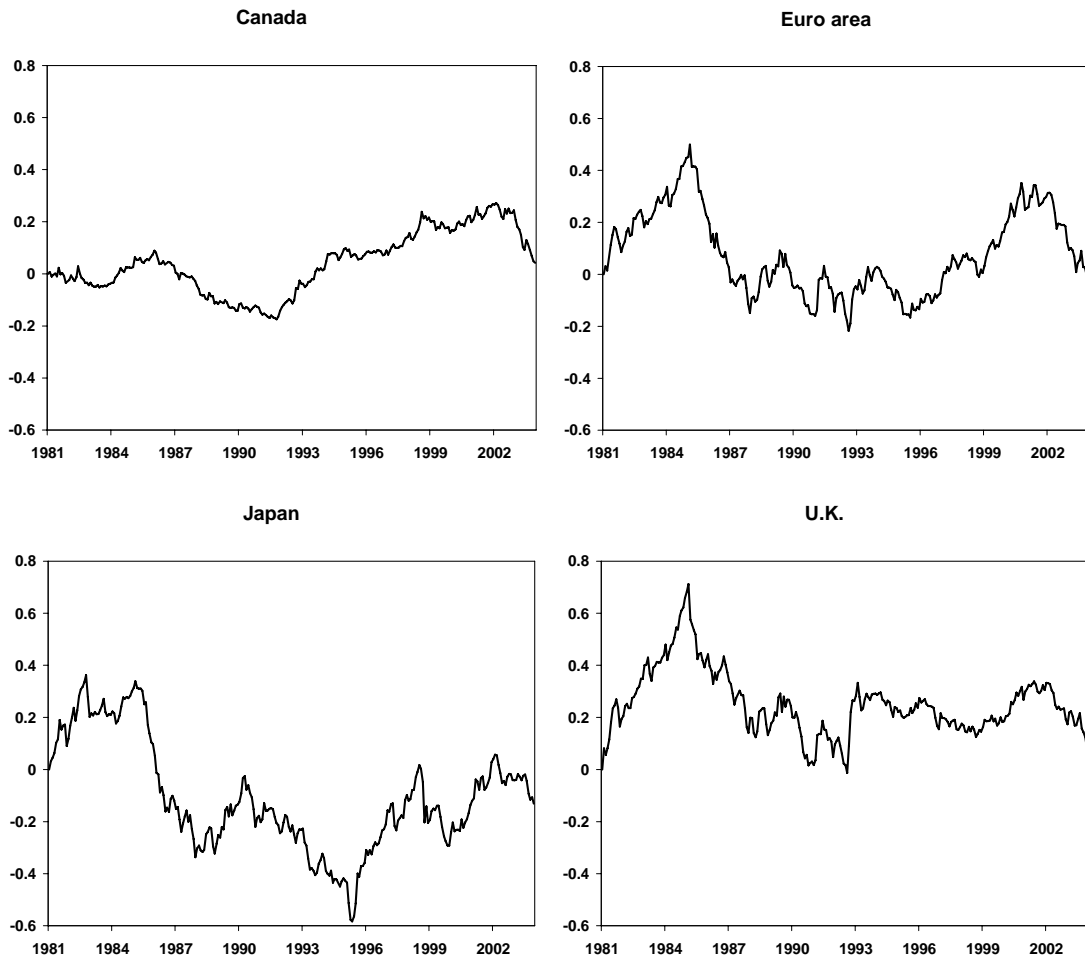
$$R_{i,t} = \alpha_i + \beta_i R_{i,t-1} + \sum_{k=1}^{l_i} \gamma_{i,k} \Delta R_{i,t-k} + u_{i,t},$$

where  $R_{i,t}$  is the log of the real exchange rate and the value of  $l_i$  is taken from the OLS ADF unit root test results. The empirical  $p$ -values as well as the critical values are obtained using Monte Carlo Simulations. \* and \*\* denote significance at the 10% and 5% level, respectively.

Country	$\alpha_i$	s.e.	$\beta_i$	s.e.	half-life	$l_i$	$\tau_i$ [ $p$ -value]	Wald [ $p$ -value]
<b>Canada</b>	0.0003	0.0009	0.9833	0.0078	41	16	-2.13 [0.275]	21.69* [0.064]
<b>Euro area</b>	0.0025	0.0020	0.9636	0.0098	19	13	-3.72** [0.021]	
<b>Japan</b>	-0.0026	0.0023	0.9770	0.0096	30	22	-2.39 [0.231]	
<b>U.K.</b>	0.0100	0.0038	0.9607	0.0135	17	21	-2.90 [0.149]	
10% critical value			Canada				-2.69	20.25
			Euro area				-3.15	
			Japan				-2.80	
			U.S.				-3.10	
5% critical value			Canada				-2.98	22.39
			Euro area				-3.41	
			Japan				-3.11	
			U.S.				-3.41	

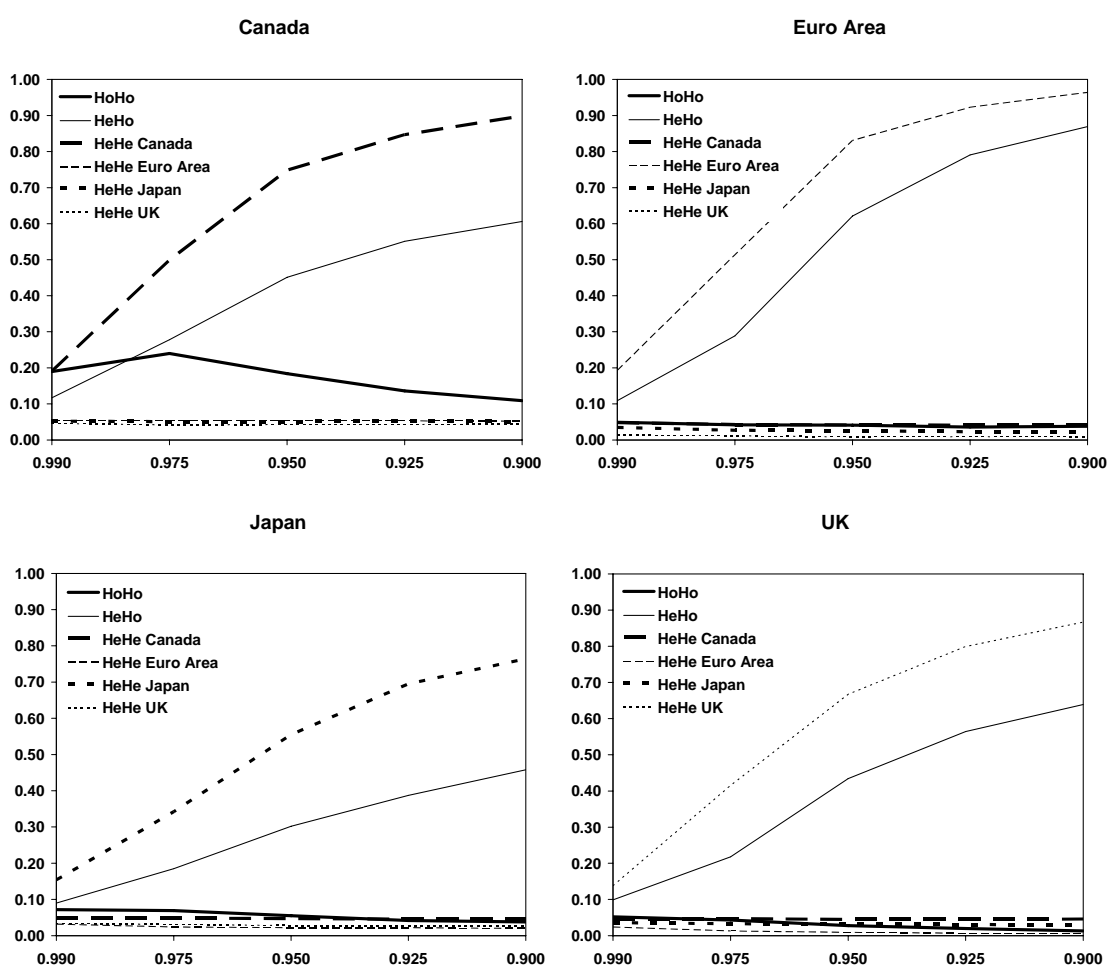
**Figure 1**  
**Real exchange rates against the U.S. dollar**

This figure presents the (log) real exchange rates of Canada, the Euro area, Japan, and the U.K. versus the U.S. for the sample period 1981:01-2003:12.



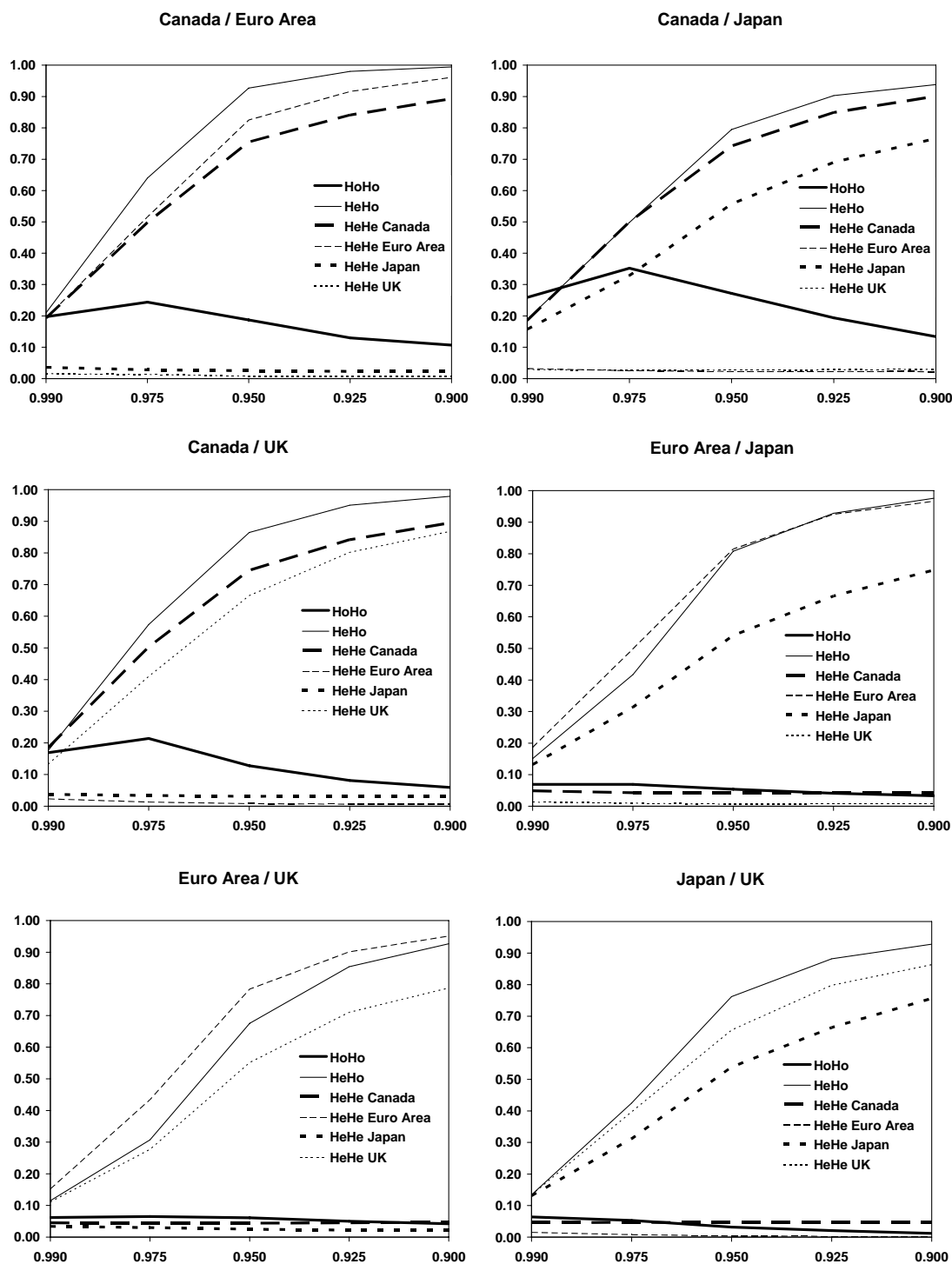
**Figure 2**  
**Estimated power functions of the multivariate methodologies (3 unit roots)**

This figure presents the estimated power functions for the multivariate methodologies in which (i) the mean reversion parameter is estimated homogeneously and the unit root test is performed for the entire sample of real exchange rates (HoHo methodology), (ii) the mean reversion parameters are estimated heterogeneously, but the unit root test is performed for the entire sample of real exchange rates (HeHo methodology), and (iii) the mean reversion parameters are estimated heterogeneously and the unit root test are performed for each individual real exchange rate separately (HeHe methodology). The power functions are computed with Monte Carlo simulations (see section 2 for a detailed description) based on the real exchange rates of Canada, the Euro area, Japan, and the U.K. versus the U.S. over the period 1981:01-2003:12. The power functions are estimated under the restriction that three of the four mean reversion parameters are equal to one, while the remaining mean reversion parameter (see chart titles) decreases from 0.99 to 0.9.



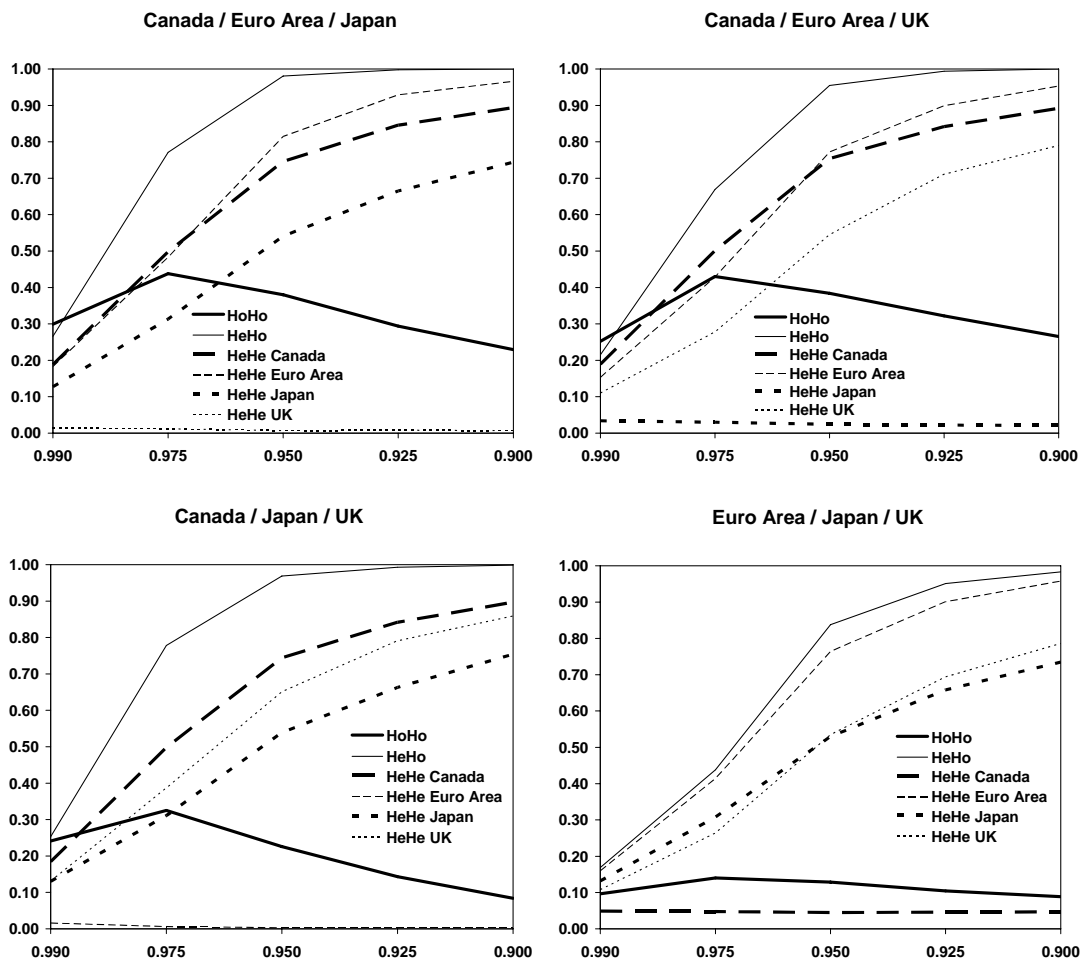
**Figure 3**  
**Estimated power functions of the multivariate methodologies (2 unit roots)**

This figure presents the estimated power functions for the HoHo, HeHo, and HeHe methodologies (described in Figure 2). The functions are estimated under the restriction that two of the four mean reversion parameters are equal to one, while the remaining parameters (see chart titles) decrease from 0.99 to 0.9.



**Figure 4**  
**Estimated power functions of the multivariate methodologies (1 unit root)**

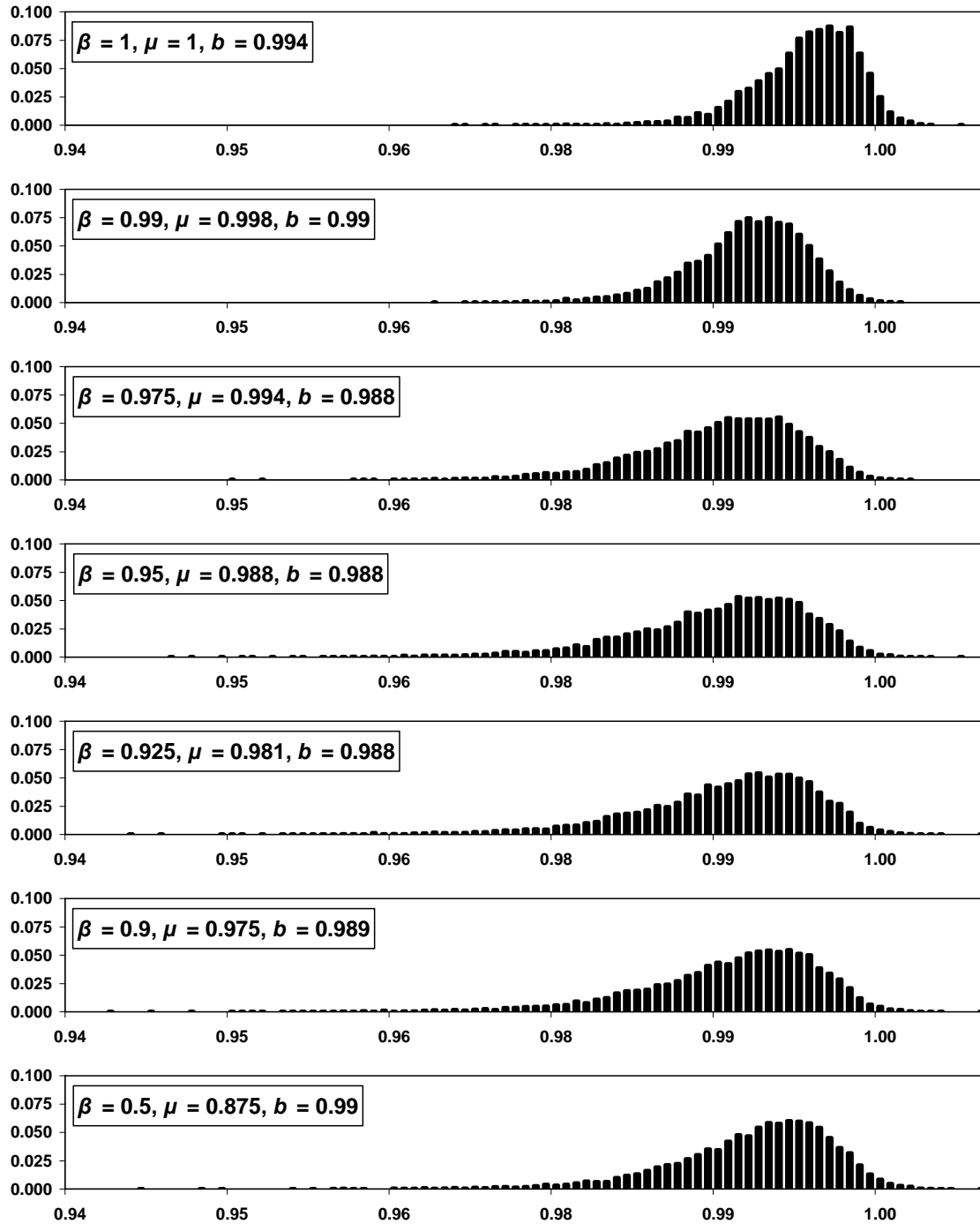
This figure presents the estimated power functions for the HoHo, HeHo, and HeHe methodologies (described in Figure 2). The functions are estimated under the restriction that one of the four mean reversion parameters is equal to one, while the remaining parameters (see chart titles) decrease from 0.99 to 0.9.



**Figure 5**

**Histograms of homogeneously estimated mean reversion coefficient (3 unit roots)**

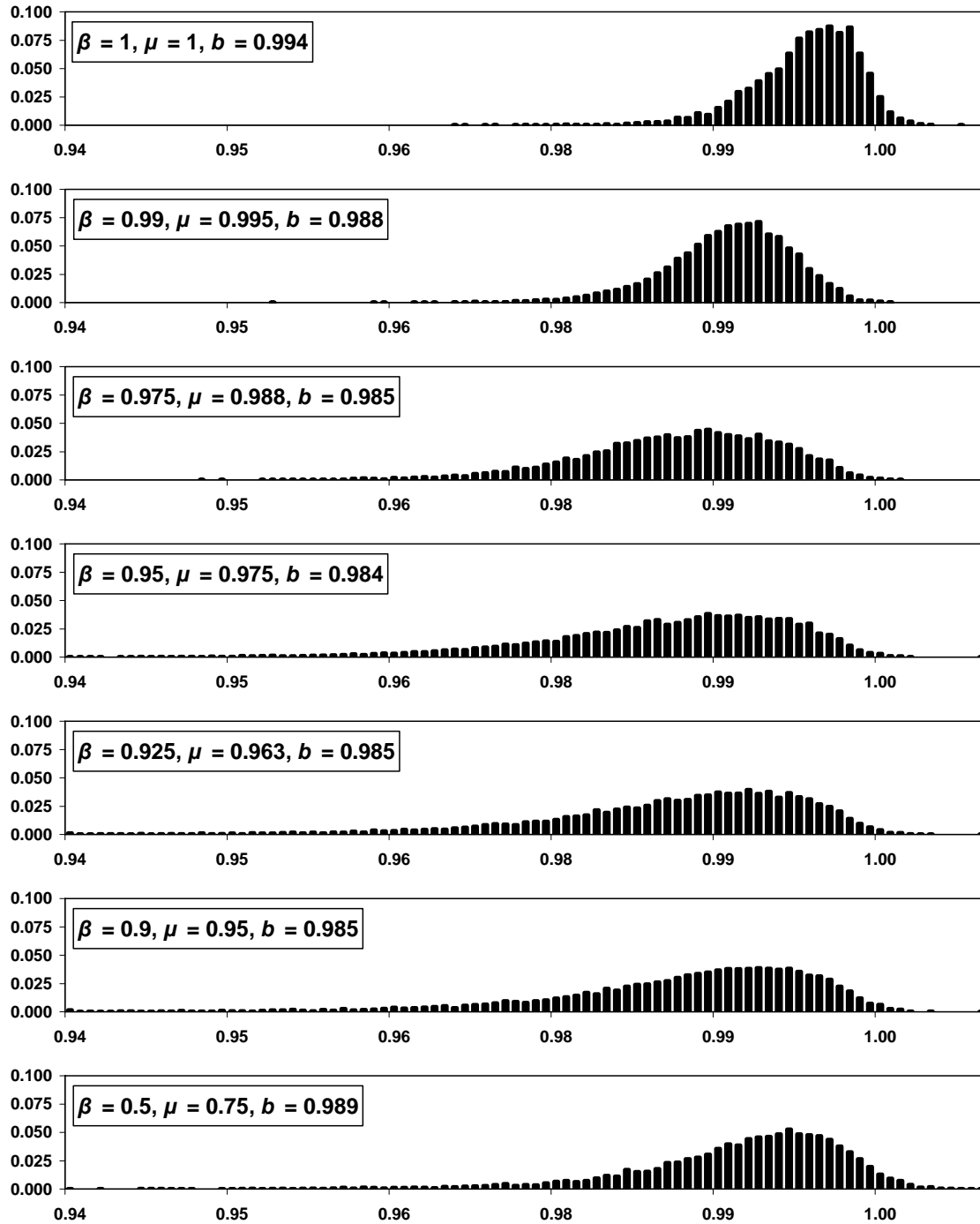
This figure presents histograms of the estimates of the common mean reversion coefficient under the HoHo methodology when the true mean reversion parameters for the set of countries {Canada, Euro area, Japan, U.K.} equal  $\{\beta, 1, 1, 1\}$ , where  $\beta$  decreases from 1 to 0.5. In the histograms,  $\mu$  represents the average of the true mean reversion parameters, while  $b$  denotes the (homogeneously) estimated mean reversion parameter.



**Figure 6**

**Histograms of homogeneously estimated mean reversion coefficient (2 unit roots)**

This figure presents histograms of the estimates of the common mean reversion coefficient under the HoHo methodology when the true mean reversion parameters for the set of countries {Canada, Euro area, Japan, U.K.} equal  $\{\beta, 1, \beta, 1\}$ , where  $\beta$  decreases from 1 to 0.5. In the histograms,  $\mu$  represents the average of the true mean reversion parameters, while  $b$  denotes the (homogeneously) estimated mean reversion parameter.

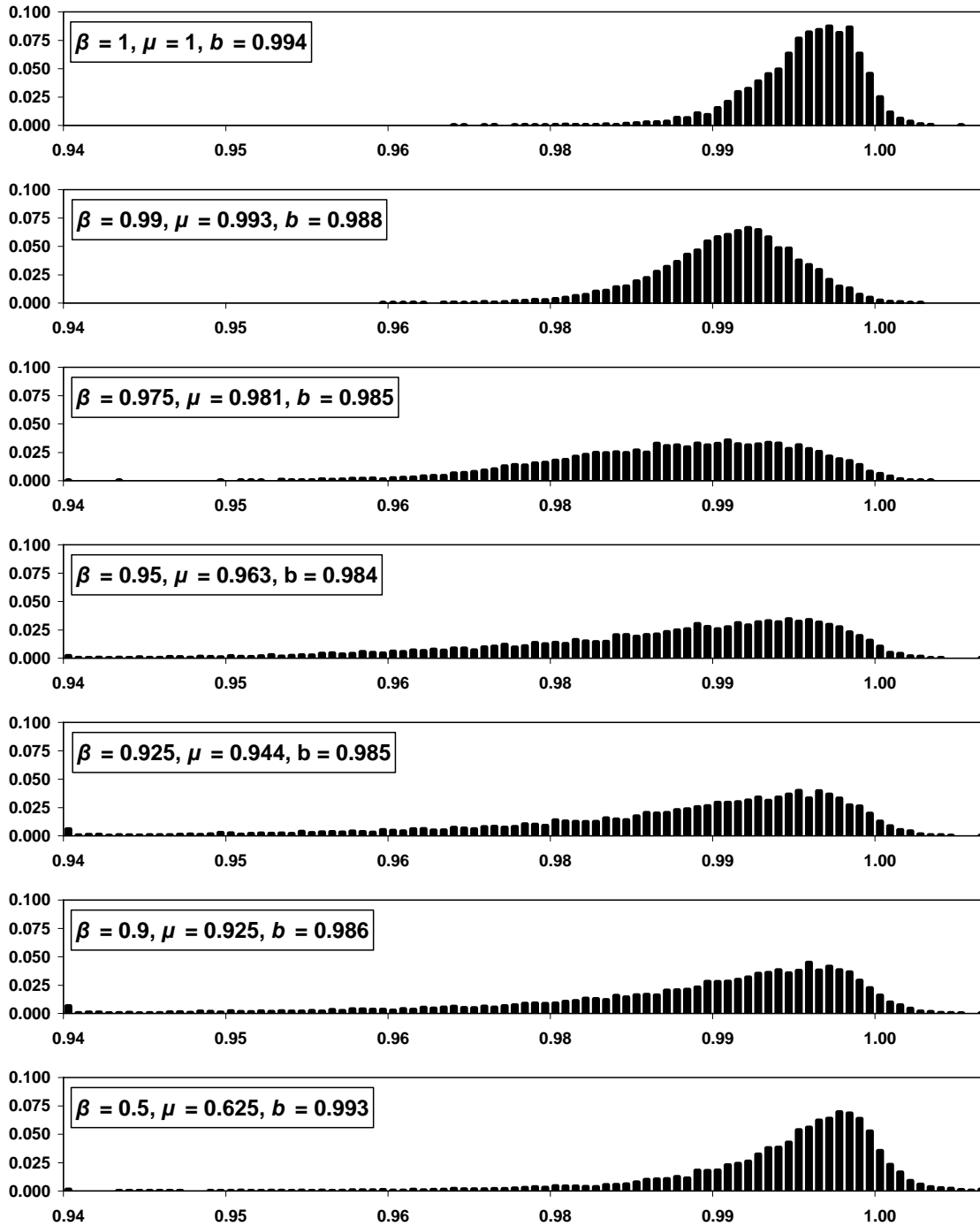




**Figure 7**

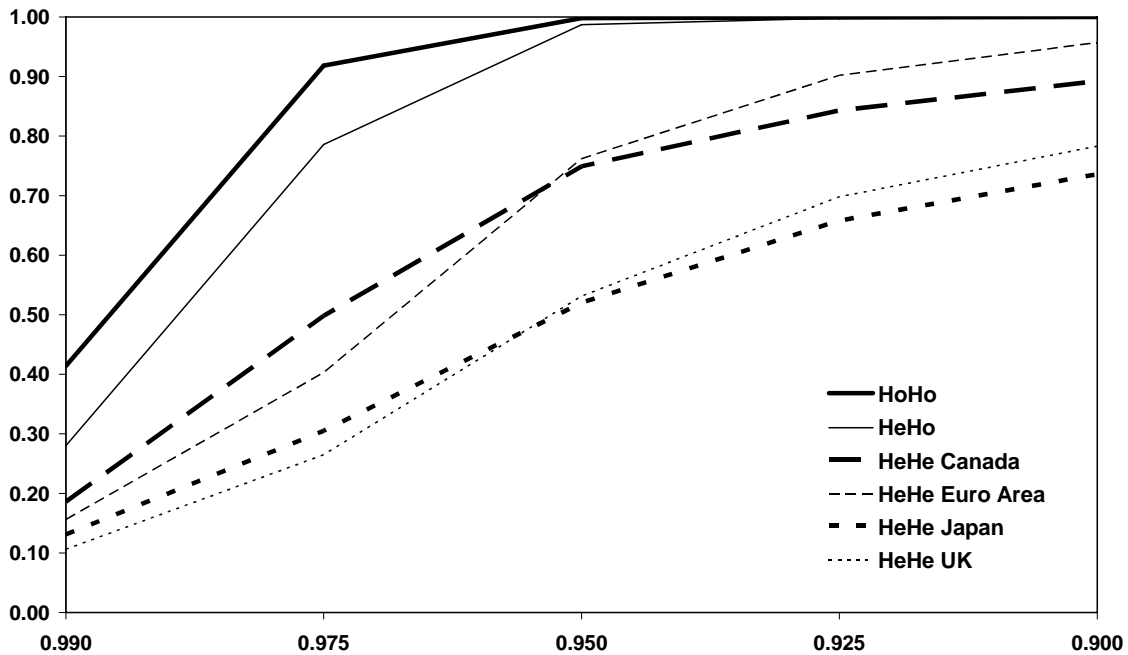
**Histograms of homogeneously estimated mean reversion coefficient (1 unit root)**

This figure presents histograms of the estimates of the common mean reversion coefficient under the HoHo methodology when the true mean reversion parameters for the set of countries {Canada, Euro area, Japan, U.K.} equal  $\{\beta, 1, \beta, \beta\}$ , where  $\beta$  decreases from 1 to 0.5. In the histograms,  $\mu$  represents the average of the true mean reversion parameters, while  $b$  denotes the (homogeneously) estimated mean reversion parameter.



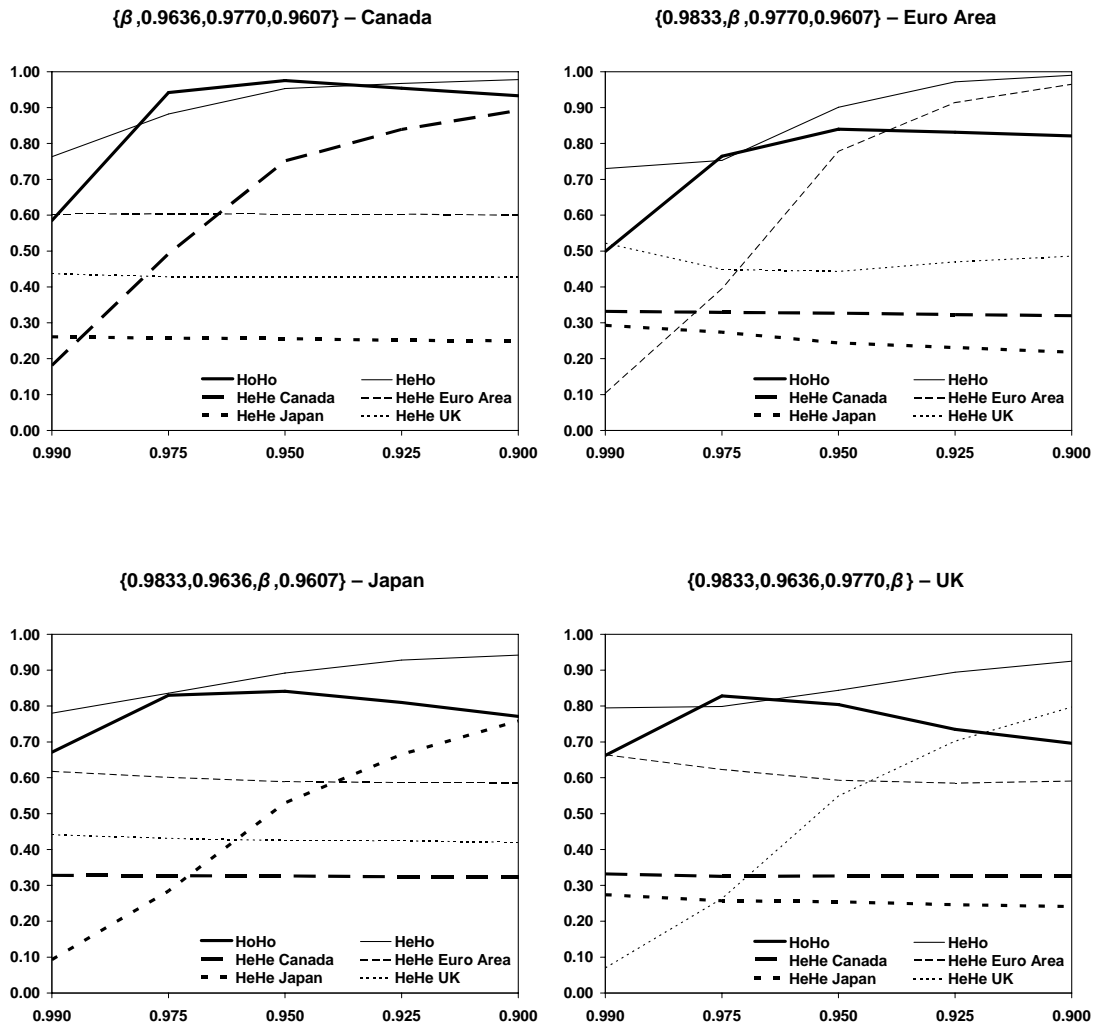
**Figure 8**  
**Estimated power functions of the multivariate methodologies**  
**when mean reversion is homogeneous**

This figure presents the estimated power functions for the multivariate methodologies when the true parameter values for the set of countries {Canada, Euro area, Japan, U.K.} equal  $\{\beta, \beta, \beta, \beta\}$ , where  $\beta$  decreases from 0.99 to 0.9. The figure depicts power functions for the multivariate methodologies in which (i) the mean reversion parameter is estimated homogeneously and the unit root test is performed for the entire sample of real exchange rates (HoHo methodology), (ii) the mean reversion parameters are estimated heterogeneously, but the unit root test is performed for the entire sample of real exchange rates (HeHo methodology), and (iii) the mean reversion parameters are estimated heterogeneously and the unit root test are performed for each individual real exchange rate separately (HeHe methodology). The power functions are computed with Monte Carlo simulations (see section 2 for a detailed description) based on the real exchange rates of Canada, the Euro area, Japan, and the U.K. versus the U.S. over the period 1981:01-2003:12.



**Figure 9**  
**Estimated power functions of the multivariate methodologies (no unit roots)**

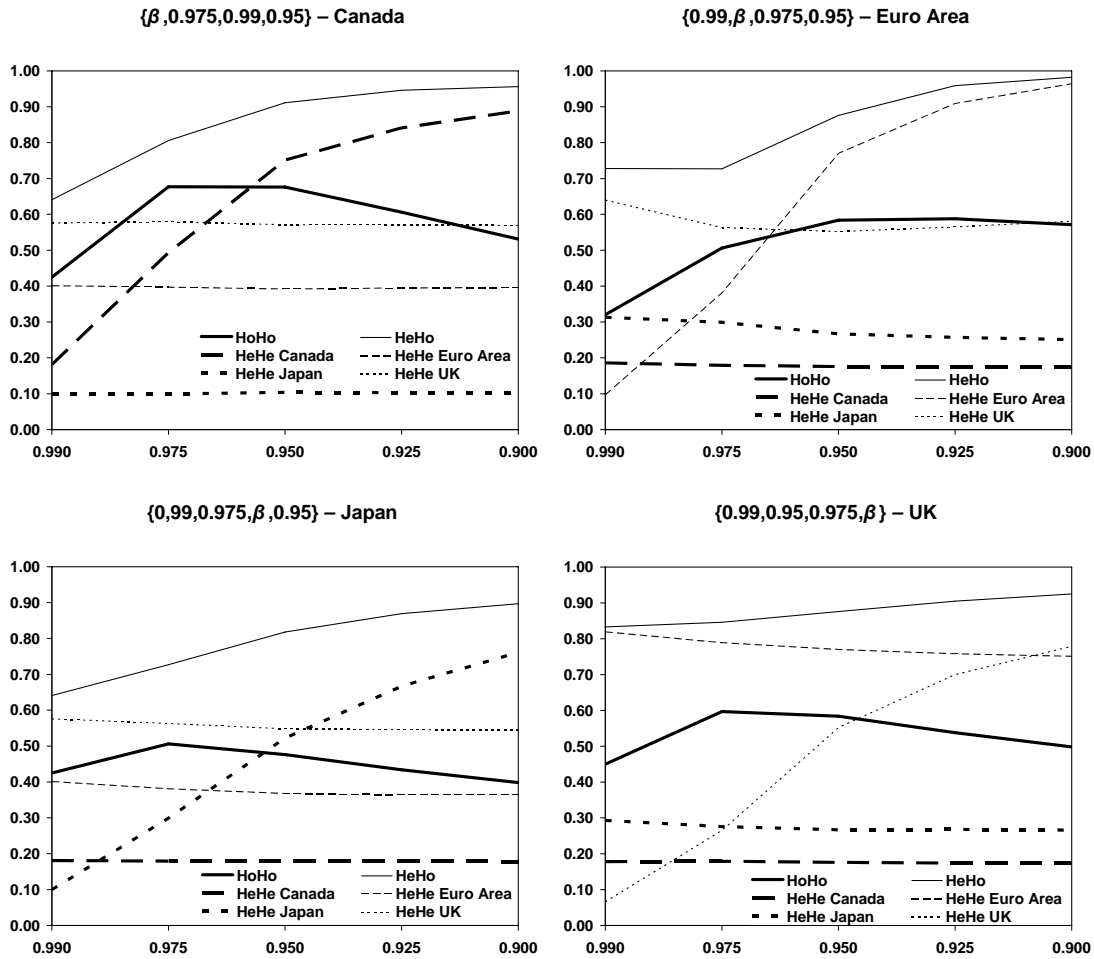
This figure presents the estimated power functions for the HoHo, HeHo, and HeHe methodologies (see description Figure 2). The functions are estimated under the restriction that three of the four mean reversion parameters are fixed at a level smaller than one (set equal to the estimated values in the heterogeneous SUR estimation), while the remaining parameter (see chart titles) decreases from 0.99 to 0.9.



**Figure 10**

**More estimated power functions of the multivariate methodologies (no unit roots)**

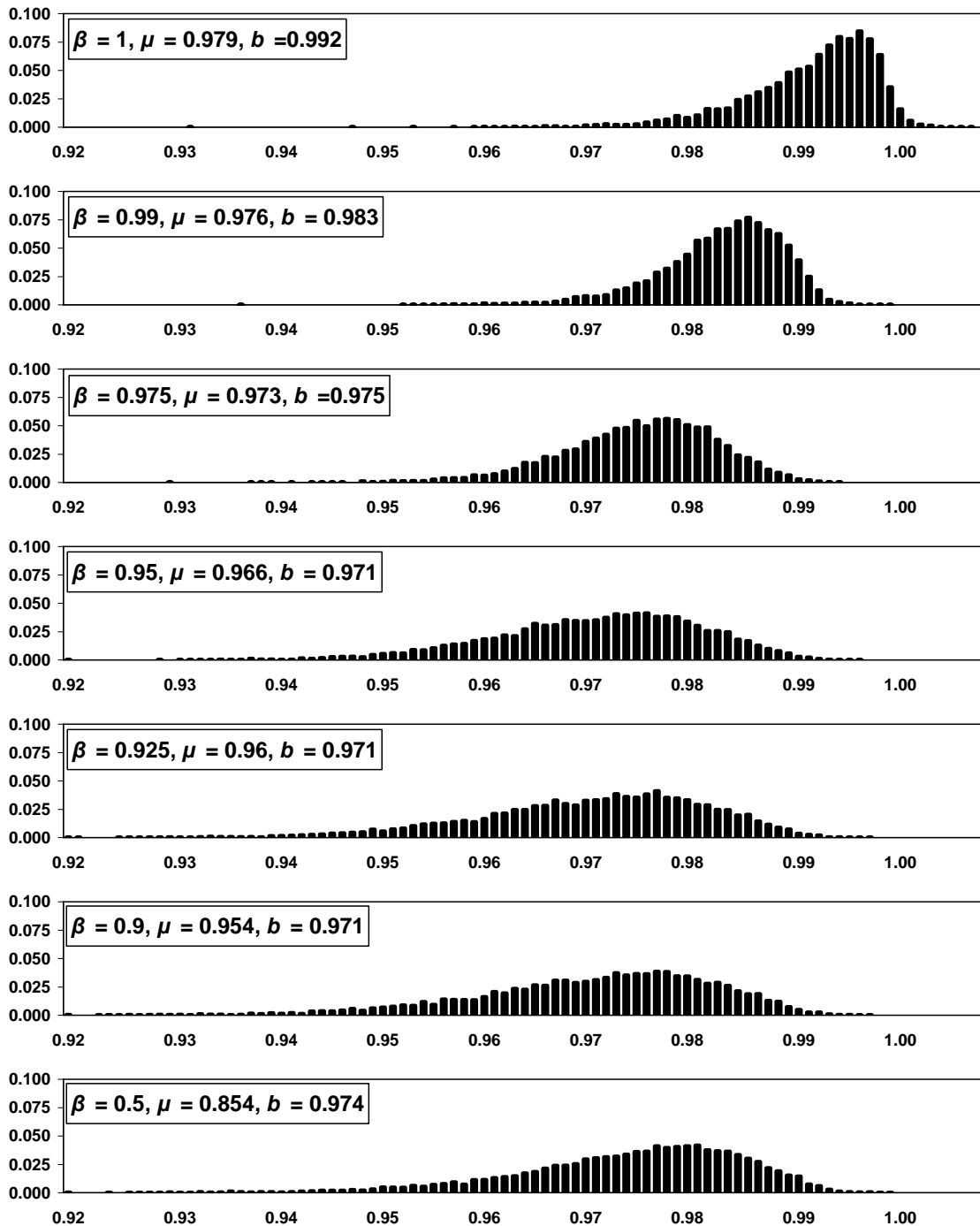
This figure presents the estimated power functions for the HoHo, HeHo, and HeHe methodologies (described in Figure 2). The functions are estimated under the restriction that three of the four mean reversion parameters are fixed at a level smaller than one (chosen from the set  $\{0.99, 0.975, 0.95\}$ ), while the remaining parameter (see chart titles) decreases from 0.99 to 0.90.



**Figure 11**

**Histograms of homogeneously estimated mean reversion coefficient (no unit roots)**

This figure presents histograms of the estimates of the common mean reversion coefficient under the HoHo methodology when the true mean reversion parameters for the set of countries {Canada, Euro area, Japan, U.K.} equal  $\{\beta, 0.975, 0.99, 0.95\}$ , where  $\beta$  decreases from 1 to 0.5. In the histograms,  $\mu$  represents the average of the true mean reversion parameters, while  $b$  denotes the (homogeneously) estimated mean reversion parameter.



**Figure 12**

**Histograms of heterogeneously estimated mean reversion coefficient (no unit roots)**

This figure presents histograms of the estimates of the heterogeneous mean reversion coefficients under the HeHo and HeHe methodologies when the true mean reversion parameters for the set of countries {Canada, Euro area, Japan, U.K.} equal  $\{\beta, 0.975, 0.99, 0.95\}$ , where  $\beta$  decreases from 1 to 0.5. In the histograms,  $\mu$  is the average of the true mean reversion parameters, while  $b$  denotes the (heterogeneously) estimated mean reversion parameter.

**Canada**

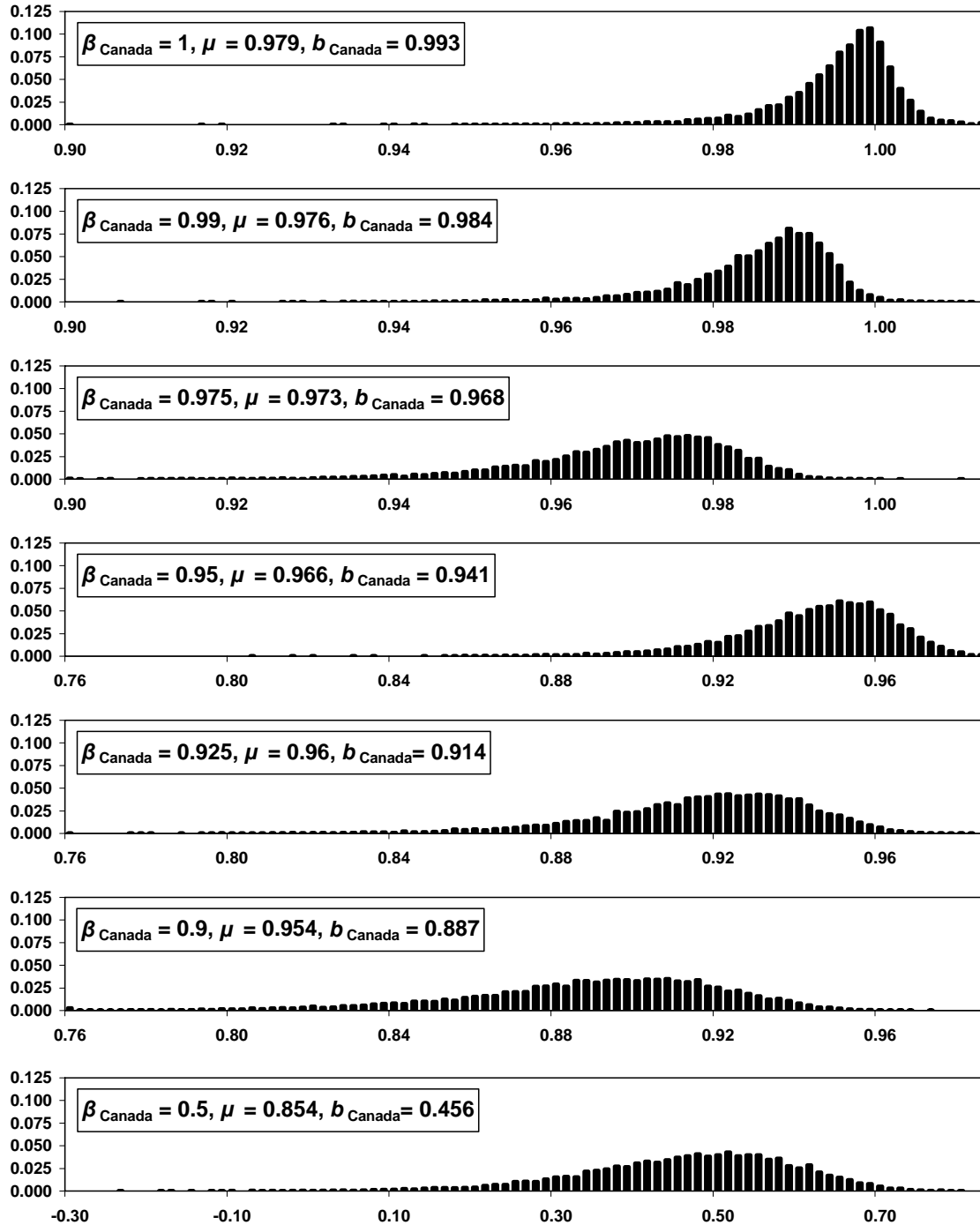


Figure 12 – continued

Histograms of heterogeneously estimated mean reversion coefficient (no unit roots)

Euro area

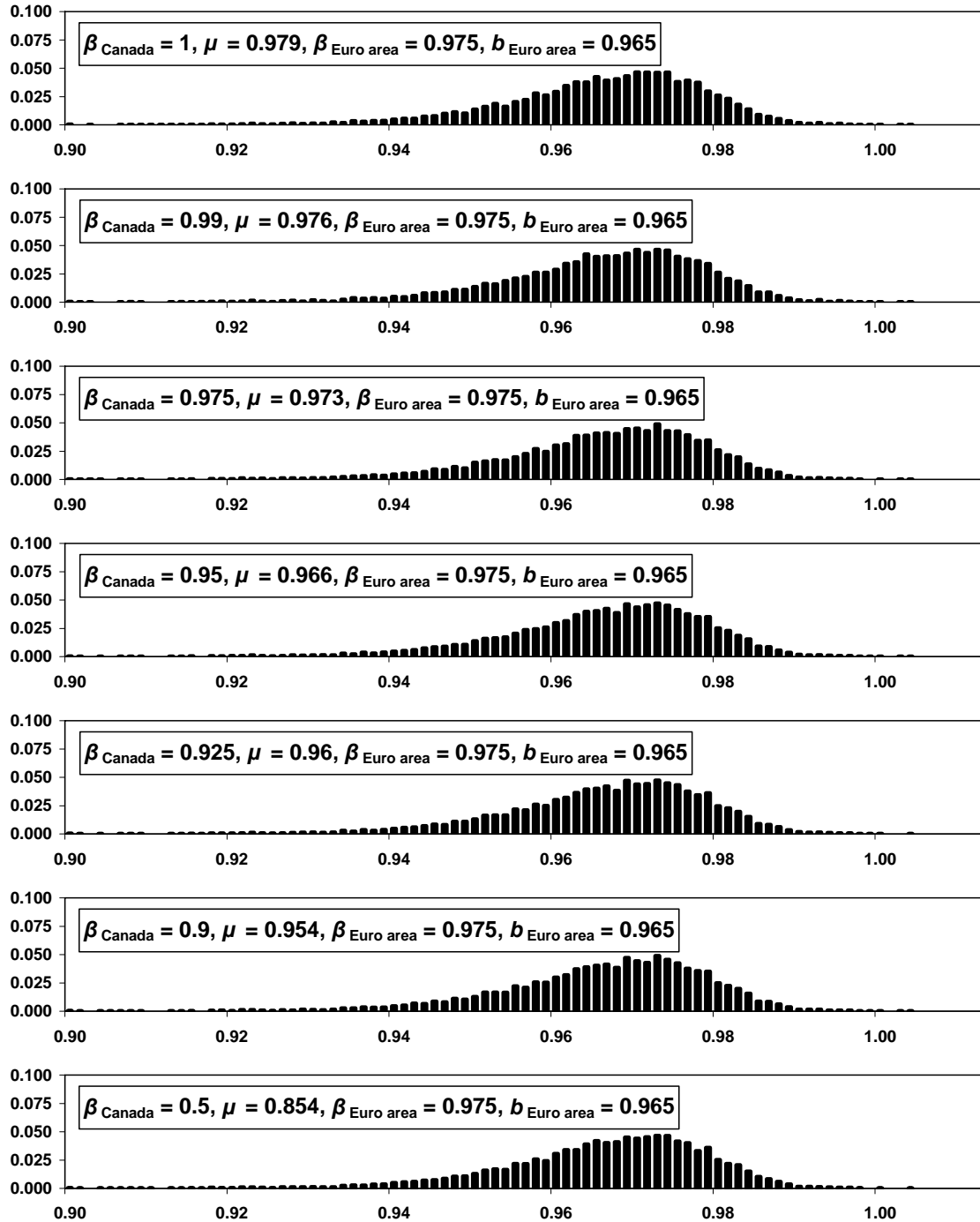


Figure 12 – continued

Histograms of heterogeneously estimated mean reversion coefficient (no unit roots)

Japan

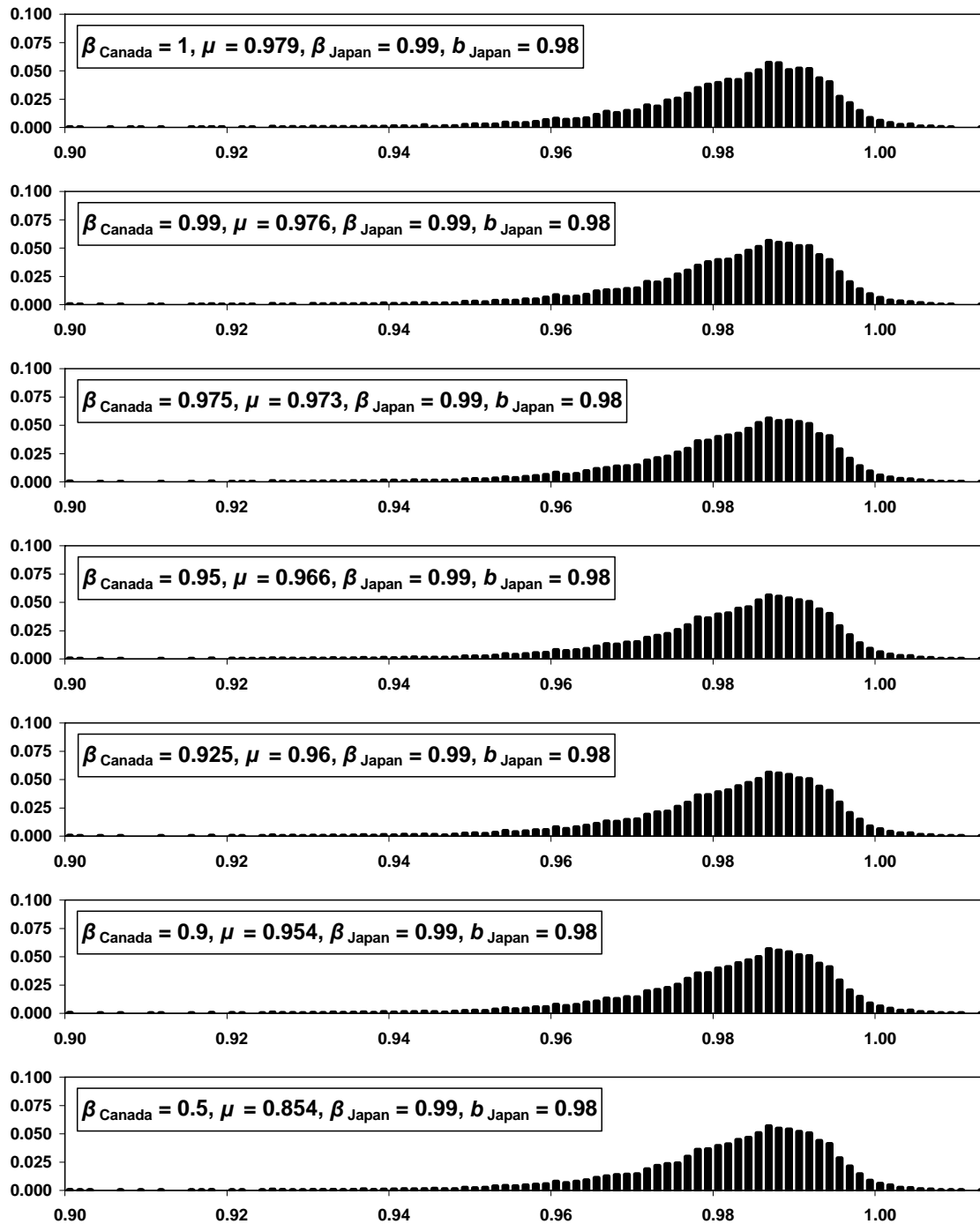




Figure 12 – continued

Histograms of heterogeneously estimated mean reversion coefficient (no unit roots)

U.K.

