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Introduction to Quantum Mechanics and Objectivity

Patrick A. Heelan
Georgetown University, heelanp@georgetown.edu

CHAPTER ONE

INTRODUCTION

SECTION I: METHODOLOGICAL INTRODUCTION

Intentionality and World

The *intentionality structure* of an act of knowledge is the orientation of a human knowing *(noetic)* subject towards a horizon of knowledge constituted by a certain ordered context of objects given or to be given in experience. The empirical answer to a particular noetic orientation on the part of a human subject constitutes a *noema.*

The total ordered context of all actual or possible objects is called a *World.* It is the "horizon of all horizons" [1]. Kant took the notion of World to be a regulative idea or principle [2]. We do not accept this view. Nor do we accept the view that the World is a derivative notion secondary to the objects it contains and a mere totalization of these objects. The World is prior to its elements; it gives reality to its elements. The World may be considered rather as the broad field of human activity which as far as the active and inquiring person is concerned is presupposed by the activity of doing and questioning. It is part explored and part mystery. But the part which is mystery is not totally hidden. It is foreshadowed in outline as the full domain which human empirical activity can attain.

The noetic intention is an attitude of inquiry, of questioning attention to what is given in experience, accompanied by an active search for what is already foreshadowed in some way by the question even before any reply is obtained from the World. A noetic intention then constitutes a reality-outline to be filled, and the filling of that

1 A. de Waelhens, *La Philosophie et les expérience naturelles* (The Hague, Nijhoff, 1961), p. 110.

2 I. Kant, *Critique of Pure Reason,* trans. by Norman Kemp Smith, (London, Macmillan, 1963), p. 392.

outline does not occur all at once, but is a progressive process in which there are always more questions at any stage than there are answers. Hence, while noetic intentions are invariant elements in the structuring of the World, the World is not a static ensemble of noemata given once and for all, but it is an organically growing system which evolves and develops according to the special intentionality-laws which rule it. The intentionality structure of a particular question then prefigures the kind of answer it will receive; not, however, that the question determines that there should be an answer, but that an answer, if one should be given, will appear not as totally disconnected with reality but as a looked-for reality within an already ordered context which we called a World, which is the horizon of the horizons of all empirically answerable questions. As M. Dondeyne says: "Consciousness is essentially *intentional;* ... it presents the form of a dialectical relation between a 'noesis' and a 'noema', the two calling to each other, and constituting one another in an indissoluble unity ... If an intention is to be actualised, it must be incarnated in a behaviour *'sui generis'* called 'observation' ... ; for example, if a countryside seems to be dark and sad, it is in part because I am sad discouraged; but it is also equally true that sunless weather contributes to my sadness" [1].

A *noema* is an object of concrete factual knowledge. It is affirmed by a knowing subject as "real" - not in isolation from the rest of reality - but precisely because it partakes in the reality of a total ordered context of actual and possible realities which is his World. True reality, then, for a subject is his World. It constitutes the horizon in which single isolated events have a place if and only if they are real.

It follows from this that there are many Worlds. Each epoch of human history and each epoch of one's own personal history has its World. Childhood, youth, maturity and old age have their Worlds, different perhaps for different people. There is the World of the physician, the World of the sportsman, the World of the husband, the World of the wife. We shall be concerned with the World of the physical scientist in the twentieth century.

Each of these Worlds represents some subject's sphere of reality; but to see it as such, and to explore the richness of the reality revealed in its perspective and illuminated by its light, one must be placed at the noetic pole of such a World. The failure to do so leaves the World an

[1] A. Dondeyne, *Foi chrétienne et pensée contemporaine*, (Louvain, Publications Universit., 1961), pp. 25-26.

incoherent jumble of pseudo-facts, distortions, and "abstractions" [1]. How often have we not heard humanists and philosophers mock at the scientific culture of our day, while remaining profoundly ignorant, not only of its depth, complexity and articulation, but also of the human motivations of the scientists themselves. On the other hand, scientists have on the whole little sympathy with a humanism which often speaks pityingly of the agony of the human condition but with such self-pity that no energy is left to better it. C. P. Snow has described with a touch of bitterness this polarisation of our culture between two hostile Worlds: "The great edifice of modern physics goes up", he wrote, "and the majority of the cleverest people in the modern world have about as much insight into it, as their neolithic ancestors would have had" [2]. If, then, it is our aim to explore the reality structure of modern physics, a necessary condition of this is that we learn to place ourselves sympathetically at the noetic pole of perspective of a working scientist. To fill this position, we have chosen one of the creators of modern physics who, because of this, is also one of its most authentic interpreters, namely, Werner Heisenberg. He will be our guide to the World of quantum physics and the interpreter to us of its reality.

A World is, at least in some way, *given in and through experience.* Husserl defined it to be the "totality of objects that can be known through experience, known in terms of orderly theoretical thought on the basis of direct present experience" [3]. The objects in question were for Husserl "given primordially in perception". It is our intention to enlarge the notion of World to include the horizon of objects known through the interpretation of data. Though not given "primordially in perception" these interpretative objects, like atoms, electrons, etc., are none the less given through experience, and constitute an extension of the notion real. They comprise a total ordered context of objects, whose "reality" is based upon the interpretation of sensible signs which reveal to the inquiring mind of the scientist the presence of these objects in an experimental situation. The structure of this new World of hidden objects revealed through sensible signs will be investigated in the course of this dissertation.

1 "The properties of a physical theory are formulated in abstract mathematical language. Let us compare them with a musical score. For those who cannot read notes, the musical score is dead, but the man who understands them hears the melody in them". C. F. von Weizsacker, *The World View of Physics* (London, Routledge and Regan Paul, 1962), p. 35.

2 C. P. Snow, *The Two Cultures and a Second Look* (Cambridge, 1964), p. 15.

3 E. Husserl, *Ideas* (London: 1931), pp. 51-2. Cf. also A. de Waelhens, *op. cit.*, pp. 107-121, *Le Monde*.

A World is also intersubjective. It is a public arena in which many people meet. People meet by orienting themselves mutually to one another in a common World or in the ground common to their Worlds. This overlapping of Worlds is a condition *sine qua non* of conlmunication between people. The range over which they can communicate, and the extent to which they can be in contact, is determined by how much of a common World they share. A World, then, is essentially a meeting place of a group – of young or old, of philosophers or scientists. It is a condition of cooperative science. Moreover, since the inquiring mind of man is never still but ceaselessly tries to unveil more and more the potential riches of reality, the World itself is also affected by the scientific enterprise.

Objects and Objectivity

If the true home of real objects is a World, and if a World is constituted by publicly accessible objects, how are these to be described? Is there one kind of public object or are there many kinds? Does the kind of object affect the meaning of "reality"?

In the first place, there are two kinds of *public objectivity:* one is the public objectivity of the *idea* (or *concept),* and the other is the public objectivity of a *reality.* The former is the property of whatever has an exact and precise definition independently of particular places, times and factual judgements. It belongs, not to any World, but to the realm of ideas. The latter, however, makes its,appearance in a World of real things, as the object of factual judgements, founded upon concrete empirical experience; and hence its description contains an irreducible element of the imprecise and indeterminate. Whatever can be precisely and determinately defined by us is not as such a reality but an idea.

In the second place, let us describe three classes of objects, and give names to the *in-itself* correlate of each, viz., the correlate of each which transcends consciousness.

The first is an object which is a unity, identity, whole and the stable subject of properties, and which may be either an object given in perception (viz., a phenomenal object) or a constructed object – like an electron – which is linked by us to existence through sensible signs. The transcendent being correlated with this object is called by us a *thing* [1].
This first class contains the following class as a sub-division.

The second class is that of phenomenal objects. This is the class of objects "given primordially in perception". It might be described as a

[1] We are using "transcendent" in the Kantian sense of "noumenal" or "in-itself".

stable subject of perceptible properties in a spatially organised World. In so far as this is represented in consciousness, we shall call it a phenomenal object; in so far as it transcends consciousness we call it a *body in the strict sense.* Allied to the notion of body as the transcendent correlate of a phenomenal – and therefore perceptible – object, there are two limiting concepts which we shall include under the name *body.* They are: (I) what is conceived to have determinate spatial coordinates at each instant – as, for example, a classical particle – even though it is not perceptible (provided it is capable of yielding some sign of its presence); and (2) a *field* as an infinitely extended medium for three-dimensional wave motions (provided also that it is capable of yielding a sensible sign of its presence).

The kind of objectivity which is found here is one based upon the *exteriority of subfect and object in perception, and we call it empirical objectivity.* This may be subdivided into *phenomenal objectivity* (for a phenomenal object) and *bodily objectivity* (for a body). This kind of object, however, is not so constituted by the act of knowing that it is entirely separated from or independent of all subjectivity; for exteriority implies its correlate, viz., the interiority of a subject, from which it cannot be divorced. It is, then, always an object-for-me.

Is it possible for a knowing subject to know itself objectively? It is evident that a contradiction would arise if we were to state that within the relation of objectivity just described, the subject could become object. However, there is a kind of objectivity in which even the subject as such can participate; that is an objectivity in which the object is constituted as simply independent of a relation to a subject: this we call *formal objectivity.* It belongs to whatever is affirmed as a virtually unconditioned object on the basis of evidence. In physics, the evidence is provided by a process of experimental verification. This kind of object we call an *object in the strict or formal sense,* or sinlply a *strict object;* for its *intention* is simply to express *what is,* independently of the act whereby I know it as an object-for-me. The transcendent correlate of an object in the strict of formal sense is, evidently, an *individual existing being,* or a *law of being.*

Subjectivity

We define *subjectivity* to be the absence of the corresponding kind of Objectivity. *Subjectivity* then is a word with many meanings which are differentiated by the different types of objectivity defined and distinguished above.

Reality and its Criterion

Let us distinguish, moreover, the *meaning* of "reality" from the *criterion* of reality. The first defines what is meant by the term; the latter is that on account of which a thing is said by a certain knower to be real: in our case, it is a sign through which its reality is manifested to us.

It is clear that the word "reality" will have as many meanings as there are different kinds of objects which can be conceived to constitute a World. For example, if a World is conceived in the naively realist sense to be an organisation of bodies in the strict sense, then "reality" will mean "whatever can be perceived as a body". It is the characteristic empiricist understanding of the term, where meaning and criterion are scarcely separated. If, taking a more sophisticated view, the objects constituting the World are expressed by the limiting concepts of classical particle and classical field, then "reality" will mean "whatever has determinate spatial coordinates at every instant, or whatever is an infinitely extended medium for three-dimensional wave motions"; sensibility merely providing the presentative sign of their presence. Interpreting the latter condition as merely a criterion of reality, then the first part of the definition gives the characteristic rationalist meaning of the term presupposed by classical physics.

Our own view is that an ontological World is constituted only by an ordered context of *objects in the formal sense*, that is, of such objects as are affirmed as virtually unconditioned obiects – i.e., as *beings* – on the ground of evidence provided by a critical scientific process of testing and verification.

We hold, moreover, that the criterion of physical reality for us is extrinsic to its meaning, since we have no intellectual intuition of physical reality. Our view then of physical reality can be summarised in the two following points: (1) "reality" means "an object taken in the strict or formal sense within the ordered context or horizon of such objects which constitutes a World"; and (2) the criterion of reality for us is a manifestation of its presence in the World through sensible signs. This last is a rational criterion and not a purely sensible one, since the recognition of the significance of the sensible sign is a rational and not a purely sensitive act.

The World of Modern Physics

Among the many different Worlds, each defining reality for some subject, one interests us in particular, viz., the World of twentieth-century physics. How is one to investigate the reality structure of the World of modern physics? M. Dondeyne, I think, has given us the clue. Stressing the correlation between "*noesis*" and "*noema*" in science, he writes: "If the scientific object is to reveal itself to human consciousness with the structure which belongs to it, it must be approached with a scientific attitude; it must be *questioned* scientifically; that is, one must go out to meet it with hypotheses and verify these hypotheses *in* the object; that is why science – even empirical or positive science – is not the result of a purely passive attitude towards the world, but it is (something to be done' in the strict sense of the term" [1].

If science is something the scientist does, then the method of investigating the reality structure of modern physics is not to look *out there* at things in the naively realist sense of the *natürliche Einstellung* of Husserl in the hope of seeing electrons, protons, etc., but to reflect on the noetic intention of the scientist, to see what kind of objects he was looking for, and by criticising this to arrive at a correct notion of the ontological content of physics. The scientist has unveiled by his experimental activity new but shadowy physical objects. Do they belong to the scientist's World of reality in the same way as do the tools and instruments of his research? Does scientific methodology imply a certain meaning of "real"? Is it necessary that scientists have a comnlon meaning for "reality"? The pre-philosophic (or natural) outlook of a physical scientist in post-classical physics is rarely that of naive realism. Electrons, protons, etc., make their appearance in the context of a World-out-there of bodies but they are never directly given as bodies in this World. A cursory survey of current scientific writings shows that two kinds of natural pre-philosophic outlooks prevail among scientists to-day. There is the empiricist-positivist outlook on the one hand which is content with practical results, with what works. There is the rationalist outlook on the other hand which assumes on the basis of the Newtonian tradition that only that which has well defined space-time coordinates is a reality. Only a careful analysis of scientific method and a criticism of the pre-philosophic conceptions of modern scientists will succeed in separating the true noema which is the object in the forn1al sense of physics from the intentionality-structure of the scientific method through which it is revealed.

An investigation of this sort of the intentionality structure of quantum mechanics is of interest not merely to philosophers but also

[1] Dondeyne, *loco cit.*, p. 26.

to many physicists; for many to-day are deeply disquieted by the conceptual paradoxes which lie at the foundations of quantum physics. This has led to a revival of interest in many of the old controversies and to some new ideas, but largely to a resurrection of old ones which had been forgotten. Professor Wigner sums up the situation thus: "The orthodox view [viz., of Bohr, Heisenberg and the Copenhagen School] is very specific in its epistemological implications. This makes it desirable to scrutinise the orthodox view carefully and to look for loopholes which would make it possible to avoid the conclusions to which the orthodox view leads. A large group of physicists finds it difficult to accept these conclusions and, even though this does not apply to the present author, he adnlits that the far-reaching nature of the epistemological conclusions makes one uneasy". Professor Wigner then adds the following suggestion: "The misgivings, which are surely shared by many others who adhere to the orthodox view, stem from a suspicion that one cannot arrive at valid epistemological conclusions without a careful analysis of the *process of the acquisition of knowledge"* [1]. The chapters that follow are largely a commentary on this remark.

SECTION II: PHILOSOPHICAL INTRODUCTION

Introduction

The period of crisis in physics which led to the construction of the quantum theory was viewed at the time by those intimately connected with it, not merely as a change in physics, but as a change in philosophic perspective about man, reality and human knowledge. Bohr, impressed by the difference between our everyday vision of a solid material world and the description given of it in quantum mechanics, came to the conclusion that a physicist can no longer take an uncritical attitude towards truth, reality and human knowing. All our expressions as he wrote, "bear the stamp of our customary forms of perception from the point of view of which the existence of the quantum of action is an irrationality ... In consequence of this state of affairs, even words like 'to be' and 'to know' lose their unambiguous meaning" [2]. Acknowledging the importance of clarifying at the start our basic philosophical vocabulary, it is nevertheless with great reluctance that

1 E. P. Wigner, "The Problem of Measurement", Address to the American Physical meeting at Washington, D.C., 1962; published in *Amer. Jour. Phys.*, XXXI (1963), p. 6.

2 Niels Bohr, *Atomic Physics and the Description of **Nature**,* (Cambridge: 1961), p. 19.

we propose to preface our discussion of the intentionality of quantum mechanics with what might be called a set of implicit definitions of terms. We do it with reluctance because such an attempt risks becoming a pedantic monologue, in which a series of profound problems are taken up in rapid succession and reduced to capsule formulae; for if a set of definitions is to be a useful tool– and this is the purpose of our introduction – it must incorporate in some way a definite viewpoint, implying a certain problematic and a certain tentative solution. Our excuse, then, is that such a sketch is necessary; and we wish to prefix it with an apology for seeming to treat *omnia scibilia* in a few pages.

Being and Truth

Being is what the content of any object taken in its strict or formal sense expresses or tends to express, though whether *truly or falsely* will depend on the presence or absence of certain criteria. *Truth* is the relation of conformity between the strict object of a judgement and the being represented by it. We understand this relation to mean no more and no less than what is found in the critical analysis of well-made judgements. We assume, of course, that we know from experience when we have made a well-made judgement. From an analysis of well-made judgements, we derive the conclusion that a true theory is one which is asserted to be independent of all subjectivity (i.e. independent of its being an object of knowledge) and posited in the absoluteness of being. This we have called formal objectivity, and it is constituted by an act of the mind which affirms that a sufficient set of conditions is fulfilled to provide a rational ground for the affirmation of absoluteness (or unconditionality).

The strictly real or ontologically real is the kind of being affirmed or affirmable of bodies or things, and it is expressed by the content of the strict object of true judgements in physics and in everyday life. Restricting ourselves to the subject matter of physics, we can say that, since we lack intellectual intuition of these, they are presented to us in knowledge as *conditioned* by the necessity of manifesting their reality through some criterion. An object of knowledge, then, which does not itself contain the criterion of its reality, may remain a mere thought-object to be considered or supposed (whether as a pure idea or as a phenomenal object). If, however, the criterion of reality is given simultaneously with it, the object of knowledge may be asserted as strictly or ontologically real.

The criterion for the reality of an individual factual object is that it should be given – either directly or indirectly – in perception, and recognized rationally as such. The criterion for the truth of a physical theory is contained in the elaborate process of scientific testing and verification; and it is fulfilled only asymptotically with an ever increasing probability. However, a distinction has to be made between the *criterion* and the *truth* of a theory. The criterion is generally a complex and unending set of tests, predictions and experiments which comprise an open set of conditions linked asymptotically with the truth of the theory. However, the set of conditions is not a linear chain of conditioned conditions regressing indefinitely, but a set of true factual judgements which individually and collectively provide the evidence for the physical theory. It is part of the physicist's training to know how to construct a set of strategic questions whose affirmative answer would constitute a sufficient basis to justify the affirmation of the theory as a virtually unconditioned object. By this we mean that the theory is *conditioned* by certain criteria (viz., the evidence) but that it is also *virtually unconditioned* because sufficient strategic criteria – judged by experienced scientists to be such – are present to justify this assertion. By the formula a *true physical theory*, we intend no more than what has been just described.

Three kinds of cognitive activities which have their place in the con1plete act of human knowing are of special interest for the work that follows: first of all, acts of perception or sensible intuition; secondly, acts of conceptual understanding; and thirdly, acts of affirmation or assertion. The object expressed by an act of the first kind is a body taken in the strict sense; the object expressed by an act of understanding is the content of a pure idea or concept; the object expressed and constituted by an act of affirmation is an object in the strict or formal sense. In the case of a factual judgement, this last act falls on a content which is defined by a concept and whose reality is indicated in perception.

Without going deeply into the genesis of these three kinds of acts and their articulation within one complex act of knowing, we propose to mention certain factors concerning them which are of great importance for the study we are about to make, and which are, in a sense, the philosophical frame of reference of the author [1].

[1] The elements of the philosophical analysis which follows have been strongly influenced by Bernard Lonergen's work, *Insight, A Study of Human Understanding* (London: Longmans, 1957), and the set of articles entitled "The Concept of the *Verbum* in the Writings of 51. Thomas Aquinas" written by him and published in *Theological Studies*, VII (1946), 349-392; VIII (1947), 35-79, 4°4-444; X (1949), 3-40, 359-393.

Concepts and Abstraction

The first is the nature of *conceptual knowledge* and of the act called *abstraction* in which a concept is produced [1]. A concept is an act of knowledge answering the question: What is so and so? which, as Lonergan says, is really a subtle way of asking: Of what are such and such sensible data a manifestation? It expresses a nature, which is not, however, an individual incommunicable nature, but a nature in so far as this specifies and can be shared by an ensemble of individuals. It seems to be, then, on the one hand, the highest common factor of a set, and for this reason it is said to *abstract* from all that is not common to members of the set; as, for example, from particular places and times: on the other hand, however, it is also the *production* or *construction* of an *ideal norm* with respect to which individuals can henceforth be compared as to the degree in which they conform to its rule or depart from it; as, for example, when a circle is defined as the locus of points equidistant from a fixed point called its centre. This last example also brings out an essential aspect of the ideal norm – it always expresses a *relation* between terms which are themselves mutually and implicitly defined by the relations; for example, in the case of the circle, the relation is one of distance equality between the centre and any point on its circumference.

Thus there exist two classes of theories regarding the mental operation of abstraction. We call the first the *impoverishmen.t theory ot abstraction* [2]. It assumes that we know individual cases first in their particularity and then, by a kind of comparison akin to factorial analysis, we isolate the highest common factor of the lot, and from this we form an impoverished representation valid for a class of things. This assumes that the content of the concept was actually known prior to abstraction though not as the common factor of a class of individual instances, and that abstraction is a conscious act of comparing mental contents. Against this theory, we object that no matter how many instances have been considered, others remain unconsidered and among these there may be some which would induce a modification of the content of the concept if they were known. The impoverishment theory of abstraction serves the useful purpose of helping to make *empirical generalisations* which are,

1 As we are not concerned with the different moments in the abstractive process, we are taking *abstraction* globally to signify the whole process.

2 We have taken the name from a remark made by E. Cassirer: "As long as we believe that all determinateness consists in constant 'marks' in things and their attributes, every process of logical generalisation must indeed appear an impoverishment of the conceptual content' '. *Substance and Function* (New York: Dover, 1953), p. 22.

however, only preparatory organisations of data, and a way of schematising experience in order to present this to the inquiring mind under the form of *regularities to be explained.* The explanation is achieved by the second form of abstraction.

The second theory is the *enrichment theory of abstraction,* and this starts from contrary premises. It asserts that knowledge of particular cases with which we start *does not* contain the concept, but that this is the end-product of a complicated set of steps in which the analysis of sensible data is of key importance. The preliminary steps are the formulations of hypotheses, their testing and their acceptance or rejection according to their value as *abstract or ideal norms* from which the sensible data do not systematically diverge. During this process certain data supposed to be irrelevant are left out; as, for example, particular places and times, the exclusion, however; is not made on the basis of a factorial analysis, but as a consequence of the kind of hypothesis which is projected. The *abstract norm* is expressed as a *manifold way of being related to other things or to a knowing subject.* The ideal norm then does not suppress the particular cases or exclude them from consideration in order to isolate a common "note", but it retains them implicitly as *sample terms in a relational structure* which constitutes a *systematic totality* [1]. This process then results in the positive enrichment of the knowledge of particulars, (a) by the formulation of sets of relations between them, and (b) by the definition of an ideal norm or law, viz., a mathematical equation or function, from which the sensible particulars do not systematically diverge. We might call this moment that of the formation of the *pre-philosophic scientific concept.* The final moment is the formation or constitution of the object in the strict or formal sense. This occurs when one takes cognizance of and reflects on the fact that the phenomenon so analysed is only a *symbol of a transcendent being* [2]. In this symbol the *ideal relational norm* corresponds to a formal similarity of essence; the *unsystematic departure* from the norm corresponds to an essential openness to multiplicity; and the *systematic totality* in its fulness and concreteness corresponds to the notion of World as an ordered context of noemata.

The former theory of abstraction, viz., the *impoverishment theory* of abstraction, has a long history, and traces of it are to be found in

[1] Ernst Cassirer calls the systematic totality so related an *Inbegrijj, ibid.*, p. 22.

[2] Cf., B. Lonergan, *Theological Studies*, x (1949), 3-40, especially p. 9.

Aristotle [1] and in most of the medieval philosophers, including Aquinas [2]. It is, however, especially characteristic of the philosophy of Scotus [3], and of the strong tradition which, through Ockham and the late medieval Nominalists, reached the modern era in two streams; the empiricism of Hobbes, Locke and Berkeley, and the rationalism of Descartes and Leibniz.

It is based upon the conception of the human mind as a mirror in which is formed a passive reflection of what is out there in the external world [4]. Its notion of objectivity is limited to the kind that is founded upon the relation of *exteriority,* and which we have called *bodily objectivity.* Its view of concept-formation can be described as a process of generalisation in which concepts are analysed, compared and factorised. This exclusive view of concept-formation is – in one way or another as we shall show later – characteristic of scientists and philosophers of science of nearly every school with, however, some notable exceptions; as, for example, Einstein, Hermann Weyl and others. One important consequence of this theory is that, among scientists of the classical rationalist tradition, reality in the concrete is taken to be as specific and detailed as the content of the defining concept; that is, if a scientific theory defines its concepts numerically – as, for example, mass, position, etc. – then individual physical systems are taken to have in reality, and independently of observation, precise and determinate values of these up to an infinity of decimal places. This is a view of the meaning of "reality" which we find among philosophers of a rationalist background and of physicists of the classical school. Its insufficiency lies in its failure to advert to the fact that numbers apply to sensible data which are merely *symbols tor us* of the concrete reality which they manifest.

The *enrichment theory of abstraction,* of which there are suggestions in Aristotle and Aquinas, has its advocates in modern times in the philosophies, for example, of Ernst Cassirer and Hermann Weyl[5]. We

1 Ernst Cassirer claims to find in Aristotle a classic exposition of this theory of abstraction *(Substance and Function,* pp. 4-9). However, such an opinion is not sufficiently nuanced. Aristotle, after all, was the first to introduce a special faculty of the intellect to account for the *production* of the intelligible form. It was through the development of this clue that the enrichment theory of abstraction obtained its characteristic feature, viz., of being *constructive* of intelligibility. Cf. also, *ibid.,* pp. 18-26.

2 As, for example, in the *Summa Theologica,* I, p. 85, a.I.

3 Cf., B. Lonergan, *Theological Studies,* VII (1946), p. 372.

4 For Scotus, sensibility was only the *occasion* of the formation of the mental image; for Aristotle, sensibility was *instrumental* in its formation.

5 Ernst Cassirer, *Substance and Function, loc.cit.;* Hermann Weyl, *PhilosoPhy of Mathematics and the Natural Sciences* (Princeton: 1949).

differ from the last two in seeing *two* moments in the process of object-formation. The two moments are: (a) that in which the sensible data are synthesised and the first (scientific or pre-philosophic) enrichment occurs and (b) that in which the second and final (or ontological) enrichment occurs and the strict object of knowledge is formally constituted.

Human knowledge in the course of its developnlent uses (a) as a moment to reach (b); but finding (a) difficult and laborious – as e.g., in scientific research – while its completion and fulfilment in (b) is accomplished naturally and easily, the existence of two moments in the full act of knowledge can easily be overlooked. Moreover, as the difficulty of scientific research lies in (a), it would be possible for us to agree with scientists and philosophers of science in their account of (a) without thereby accepting fully their views of knowledge and reality. Thus Ernst Cassirer and Hermann Weyl share the view that the enrichment of sensible data occurring at the stage of the synthesis of sensible data consists in the formation of an *ideal relational norm from which individual data do not systematically diverge* [1]. However, their view of the meaning and criterion of reality and of the relation between sensibility and intellect is different from ours. In the enrichment theory of abstraction, *the pre-philosophic scientific concept* is not an *apodictic norm* as a Scotist norm must logically be, but merely a *possible norm* which is verified *in the data up to a certain degree of accuracy in fact.* Since it is an ideal norm, individual cases are expected to diverge from it, but not in a way that can be defined. Moreover, individual cases are known to be random samples of the ideal norm. And finally, it expresses something absolute only in so far as this is a subject or a term of a set of relations within a systematic totality; this implies the ontological position that we know no physical thing in its absolute nature except in so far as this is a part of a World.

Deterministic (Causal) Theories [2]

The preceding analysis reveals the possibility of two kinds of physical theories: deterministic or causal theories and probabilistic theories.

[1] Cassirer, *loco cit.*, and Weyl, *loco cit.* We differ from both these authors in stressing that the ideal relational norm is derived *from* sense data and expresses the intelligibility present in these data, without being itself a sense datum. In our view, the sense data, when understood, is understood to be a symbol whose inner function and purpose is to manifest being; i.e., not merely the phenomenal being of the symbol, but the transcendent being to which it points. The metaphysical position of both Cassirer and Weyl is that of Neo-Kantian Idealism.

[2] In keeping with common usage among physicists, *deterministic* is here taken as synonymous with *causal.*

The first kind is represented by the construction of an ideal relational norm for sensible data from which individual cases do not systematically diverge [1]. Consider, for example, Newtonian Mechanics. A particle is defined as the subject of six independant phase-space variables, viz., position and momentum in each of three directions; all six are relative to a frame of reference and to a unit. The laws of mechanics define by implicit definition how these are related through their time derivatives and through force (also relative concepts) with one another [2]. All variables are described operationally through appropriate measuring processes which map them onto the number field [3]. The equations are such that, given the initial values of all phase-space variables and the form of the law of force, the state of an isolated system at any future (or past) time epoch can be calculated exactly. This kind of theory is called a *deterministic or causal theory* [4], since it allows the calculation of the future or past state of an isolated system if its state is given at an arbitrary origin of time. The isolated system in question, however, is not a real system, for all its variables – even position – are supposed to be defined with an infinite degree of precision, while data on any real system are obtainable only up to a certain degree of accuracy [5]. Hence the system described by Newton's Laws is one which is represented (or symbolized) by an idealised conceptual model; in other words it is an *ideal or abstract norm.*

A deterministic theory serves two functions: (1) it connotes a self-defining set of physical relations, and (2) it yields a set of mathematical functions, parametrised by the time, which describe how a set of ideal measure numbers changes with the time parameter. It does not directly describe an individual physical system but it compares this with a constructed norm, viz., with a set of precise mathematical functions of which it is to be regarded as a random sample of one.

1 This process may also be called *idealisation;* it is the product of enriching abstraction.

2 This is sometimes called a *constitutive definition.* For the nature of *implicit definition,* cf., D. Hilbert, *Grundlagen der Geometrie* (Leipzig: 1930); Weyl, *loc. cit.*, chap. I; and *infra,* chap. IV.

3 The relation between the implicit (explanatory) definition and the operational description is discussed below in chap. IV.

4 A *deterministic* of *causal physical system* sometimes denotes simply a classical particle or a classical field, for it is supposed that such a system is always governed by deterministic laws, even when the behaviour of the system at every instant is not fully known, as, e.g., in a classical thermodynamical ensemble.

5 Cf. M. Born, "The statistical interpretation of quantum mechanics", *Science,* CXXII (1955), 675-679; M. Born and D. J. Hooton, *Proc. Camb. Phil. Soc.*, V (1956), 52, 281. Born and Hooton show of what little significance in fact is the knowledge of precise initial conditions even for a classical system.

Probabilistic Theories [1]

A *random case* is one which is selected from a *range* of possibilities and belongs to a random set. A random set is a collective with the following properties: (I) it is a finite sample of elements, which are individual, concrete and independent instances of the *same ideal norm;* (2) and such that there exists a unique function which expresses the *ideal relative frequency of occurrence* of each of the possibilities. By *ideal* in the second context we mean that the relative frequencies of any finite sample of sufficiently large size is not *significantly different* from the ideal relative frequency. This property is also called the *ergodic hypothesis.*

This definition has the advantage that it incorporates the essential features of von Mises's classic definition without being open to the attacks arising from the infinite collectives in his explanation and the type of limiting processes he envisaged [2]. First of all, since only the relative frequencies are counted in any set, the order in which the elements arise or are considered is immaterial to the calculation. This satisfies von Mises's condition of *irregularity.* Secondly, his limiting frequencies in an infinite collective are no more than ideal relative frequencies in the sense we have just defined and can be understood and postulated independently of the limiting procedure which he described. They do not then belong to any concrete finite or infinite collective, but to an ideally constructed norm for all random sets of a given type. Finally, von Mises's condition of *convergence* satisfies our definition and is perhaps the most reasonable translation of it into mathematical language – i.e., if it should be proved to be consistent with itself.

The present author's intention is not to propose a new basis for the calculus of probabilities but rather to bring out one essential epistemological aspect of the classic concept of probability, namely, that the classic concept is itself an ideal abstract norm and not a concrete value belonging to any actualisable collective. Hence, as von Mises himself was aware, the postulation and subsequent testing of statistical hypotheses involves the same set of epistemological problems as the construction and testing of deterministic hypotheses [3].

[1] The exposition of the following paragraphs owes much to chap. IV of *Insight* by B. Lonergan, and to *Probability and Induction* (Oxford: 1949), by W. Kneale.

[2] R. von Mises, *Probability, Statistics and Truth* (New York: 1939). Cf., W. Kneale, *Probability and Induction,* pp. 150-167.

[3] "The relation of the theory of infinite collectivities and observation is ... essentially the same as in all other physical sciences", R. von Mises, *loc. cit.*, p. 125, quoted by W Kneale, *loco cit.*, p. 160.

Our definition, however, adds a new element of importance for it states what constitutes a member of a random set. It is whatever is judged to be a concrete and independent instance of the same ideal norm among a set of such independent instances. The ideal norm connotes the choice of a type of theory and of a set of initial conditions. The similarity of initial conditions is to be judged according to the *practical criteria of significance* employed by experienced physicists. Similar instances of the same ideal norm are not the same as equiprobable instances, since there is no reason why the distribution of instances in a random set need be governed by a constant probability measure. Random instances do not, however, differ significantly and systematically. Individual concrete cases which are similar instances of the same ideal norm of this sort constitute a *random set*.

Our definition, moreover, has the added advantage of explaining how random sets occur in experience and how they are related to our way of knowing. They occur as sets of individual instances which in experience exhibit *a margin of "uncertainty"* or *"error"*. This uncertainty is the same as the deviations from the mean which Laplace took to be the subject matter of probability. We ascribe them to a different cause [1]. The results of a set of experiments, judged by practical criteria to be performed under similar conditions, are generally distributed on a range of values and the relative frequencies of occurrence of the different values in the set tend in general to a limit. This limiting frequency is a new ideal norm, however it is conceived. In the classic definition it is conceived to be the limiting relative frequency within an infinite series of individual instances. In any event, whether it be defined in this way or as we suggest, it describes not the actual results of any test but a new abstract concept which is called by the physicist the *probability of occurrence* [2].

According to the view which we have expressed, probability laws arise out of the very nature of scientific knowing and are an essential complement of deterministic (or causal) theories. They depend on the latter for the definition of the variables, of the initial conditions and of the law of development or evolution of the physical system. They

1 As R. L. Ellis writes: "Mere ignorance is no ground for any inference whatsoever: *nihilo nihil", Mathematical and Other Writings*, ed. G. Walton (Cambridge: 1863), quoted by W. Kneale, *loc. cit.*, p. 151.

2 A *probability* or a *probability measure* is generally predicated of a particular value or of a particular interval in the range respectively; the distribution of frequencies in an ensemble based upon a set of probabilities is called a *statistical distribution.* An individual case considered as a sample of *one* taken from a statistical distribution is often called a *virtual ensemble.*

complement deterministic laws because they succeed in organising material which a deterministic law omits as irrelevant to its type of synthesis, namely, the distribution of variations of concrete measured data from the ideal mathematical law [1].

We shall have occasion later on to return in more detail to the points outlined above for the epistemological analysis of probability is one of the central problems for the interpretation of quantum mechanics [2].

Probability and Human Ignorance

Our epistemological position can be clarified by comparing it with the common – and classic – view that probability laws in physics are based upon human ignorance of aspects of the concrete situation due to such factors as, for example, the complexity of the situation or to the crudeness of measuring instruments. This was the view of Bernoulli, Laplace and Leibniz, and its classical exposition is found in Laplace's work, *A Philosophical Essay on Probabilities* [3]. It is the view most commonly held by physicists and philosophers of science to-day.

Let us distinguish two types of ignorance. The first type belongs to the man who sees a series of near-similar events happening but does not know enough about them individually to be able to deduce the law in the series. The second belongs to the man who knows that there is *no determinate law* in the series, viz., that the series is merely factual and nothing more. We hold that probability laws are founded upon the latter state of mind and that it is not really a kind of ignorance but a kind of negative knowledge. Because of this negative judgement, he can limit the possibly significant material of any series to relative frequencies of occurrence within the series, that is to *probabilities.*

It can be objected that if, like Laplace's demon, we had exact knowledge of the initial conditions of a physical process, we should then be able to predict the behaviour of the system and the need for merely probabilistic laws would vanish. This objection is based upon an ontology and an epistemology different from that defended by the present author. One source of this objection is the rationalist *Principle of Sufficient Reason* as, for example, understood by Laplace and Leibniz, which is intimately connected with the rationalist view of reality. Another source of this objection is a view of knowledge very

1 Cf., Lonergan, *Insight*, chap. IV, pp. 46-51.

2 *Infra*, chap. II, section VII, pp. 38-41.

3 Pierre Simon, Marquis de Laplace, *A Philosophical Essay on Probabilities* (New York: 1951), chap. II; and also W. Kneale, *loc cit.*, pp. 1-21.

like the Scotist one, according to which individual cases are thought to be known in individual concepts from which common notes are abstracted by conceptual analysis. If this were so, then each system would have to have the same infinite precision as their factorised norm.

On the contrary, we hold that our initial knowledge of particular cases is defective and potential. This is eventually enriched by abstraction with the construction of an ideal, abstract or limiting case which has the property that particular cases do not diverge systematically from it or that particular cases can tend to it but never reach it. The non-systematic element which enters into our knowledge of individual cases will be discussed more fully below and is connected with the essential function of sensibility in the acquisition of knowledge and in the formulation (or constitution) of the strict object in which alone physical reality is truly known by us [1].

It may be conceded to the objection that since we have no intellectual intuition of physical reality, there is always more in any particular case than we can ever know. However, this lack of knowledge is not simply a question of decimal places. The particularity of a physical reality does not consist in the supposed possession of an infinity of exact decimal places. It would be mistaken to assume that we approach asymptotically the individuality of a particular case by accumulating more and more of these. There is a limit, as every physicist knows, to the significance of any decimal series. The reason for this is that decimal places and number-mapping in general belong to the human manipulation of the sensible symbols through which reality is known by us. They are instruments useful to an abstractive mind like man's, but not to a non-abstractive intelligence which would know the concrete case in its particularity and within the context of a concrete pattern of relations. Not having sensibility, the non-abstractive intelligence would have no need to map these relations on a number field as we are accustomed to do.

For this reason we think it misleading to say that probability laws arise out of human ignorance. In one sense probabilities indicate an absence of comprehensive knowledge, viz., the intellectual intuition of concrete physical reality. In a more important sense, however, they are founded not upon ignorance but upon the abstractive character of human scientific knowing and represent an irreducible factor of scientific knowing. The contrary view which we oppose is connected

1 *Infra*, chap. II, pp. 30-32 and chap. V, pp. 107-9.

moreover with an ontology and epistemology which we find impossible to justify [1].

Probability of Evidence

There is another sense of *probability* which should be carefully distinguished from the former sense: it has been variously called "acceptability", "credibility", "reasonableness" etc. [2]. It is not a concept but a quality or mode of the affirmation of the judgement. It is the estimate of the connection between the *evidence* for a judgement or a theory made on the basis of the process of a scientific verification, and the *necessary ground* that the judgement or theory could be rationally affirmed in a virtually unconditioned judgement. A virtually unconditioned judgement – which is a *certain judgement* - is one for which sufficient conditions for a reasonable affirmation are known to be fulfilled in fact. If, however, the inquiry has not been pursued to a definitive conclusion, then a *probable judgement* can be made on the basis of insufficient evidence, where the probability in question is not measured in terms of ideal frequencies but in terms of how far or how near it is to the status of being *virtually unconditioned*. Judgements about probabilities in the first sense (viz., as ideal frequencies) can be *either* certain judgements *or,* if the evidence is not complete, merely probable judgements, where *certain* and *probable* here refer to the particular sense of probability discussed in this paragraph.

Summary

The method, aim and presuppositions of the present work are outlined in this chapter. The subject matter of the book is the quantum mechanics of Heisenberg. Its aim is to state and analyse the problematic called the "crisis of objectivity" or the "crisis of reality" in quantum physics. Its method is an analysis of the intentionality structure of quantum physics as Heisenberg conceived it to be and, through a critique of this, to arrive at a clarification of the problem and of its presuppositions, and eventually at a tentative solution. Section I deals with the method and aim of the dissertation; Section II defines some of the philosophical vocabulary used in the text.

[1] Cf. O. Costa de Beauregard, *Le second principe de la science du temps* (Paris, Seuil, 1963), pp. 47-49.

[2] "Acceptability" is used by W. Kneale, *loco cit.;* "reasonableness" is used by R. Braithwaite in *Scientific Explanation* (Cambridge: 1953); B. Russell uses "credibility" in *Human Knowledge. Its Scope and Limits* (London: 1948); Karl Popper uses "verisimilitude" in *Logic of Scientific Discovery* (London: 1959); R. Carnap's "degree of confirmation" serves the same purpose.