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Liquidity and Contagion in Financial Markets*

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Liquidity and Contagion in Financial Markets

Abstract

This paper presents a model on contagion in financial markets. We use a bank run framework as a mechanism to initiate a crisis and argues that *liquidity crunch* and *imperfect information* are the key culprits for a crisis to be contagious. The model proposes that a crisis is more likely to be contagious when (1) banks have similar cost-efficiency structures (clustering) and (2) a large fraction of the investment is in the illiquid sector (illiquidity). The latter is an *endogenous* decision made by the banks. It increases with (1) the prospect of the risky asset (risk-return trade-off) and (2) the fraction of patient consumers (liquidity demand).

1 Introduction

The world financial markets are nerve-racking not only because the markets collapse every now and then, but also because when one market collapses, other markets tend to follow, sometimes with no identifiable fundamental reasons! This *contagious* effect of the financial markets has been causing great concerns among both practitioners and policy makers. For investors, contagion implies that once a financial market collapses, they have nowhere to escape because other markets will follow suit. For policy makers, contagion implies that the financial markets and even the real economy of their country may collapse for reasons that are out of their control.

Concerns regarding contagion are renewed after a series of events in the 1990s. These include, among many others, the 1994 Mexican crises, the 1997 Asian crises, and the 1998 Russian crisis. Rigobon (1999) presents a serious empirical effort in trying to measure the exact degrees of contagion in these three episodes. However, contagious crises are not new concerns, they have plagued the world financial market since the beginning of time. One such example goes back to the world wide financial crises in 1907 when contagious panics came back and forth from one country to another and desperate depositors lined up to get their money out of failing banks. What is interesting to us, however, is not only how and why some a terrifying financial crisis happened, but also how J.P. Morgan saved the U.S. financial system from collapsing. On the one hand, he raised a huge stack of money using his power; on the other hand, he send his senior associates to those near-collapsing institutions to do an auditing job. For those without good fundamentals, for those whose assets were not enough to cover their liabilities, he let them fall; but for those with good fundamentals, he declared his support, paid off the panicking investors, and ended up with these firms. These investments, indeed, turned out to be very profitable. What was unique to J.P. Morgan is that he not only had the power to raise enough money to bail these failing firms out, but also had the technology or resource to do the auditing work to find out more information about his investments. Indeed, in our model, *liquidity shortage* and *imperfect information* and the two key ingredients form a crisis to be contagious.

This paper presents a model on contagion in financial markets. In all the episodes mentioned above, liquidity dried up during the crisis. We use the basic framework of Diamond and Dybvig's bank run model (1983) to capture such an liquidity crisis. In such a framework, consumers deposit all their endowments in the banks and banks invest the endowments between a liquid, low-risk technology (cash) and an

illiquid, risky project. Liquidity problem arises when all consumers in a bank decide to withdraw their money before the illiquid project matures.

In the model, we incorporate multiple banks differentiated by their banking and/or investment efficiency. Contagion is captured by the spread of runs from one bank to another. A securities market is also incorporated where banks can rebalance their portfolio for liquidity needs. A key assumption in the model is that, while banks know the true return of the investment, consumers only observe the market prices of them. The model shows that *liquidity crunch* and *imperfect information* are the key culprits for bank runs to be contagious.

The model proposes that bank runs are more likely to be contagious when (1) banks have similar cost-efficiency structures (clustering) and (2) a large fraction of investment is in the illiquid sector (illiquidity). The latter is an *endogenous* decision of the banks. It increases with (1) the prospect of the risky investment (risk-return trade-off) and (2) the fraction of patient consumers (liquidity demand).

In our model, a crisis can be initiated by either a slowdown of the economy (a low return to the investment) or a big negative shock to the cost-efficiency of some bank. Once a crisis begins, its contagion depends on the relative magnitudes of the liquidity demand by the running banks and the liquidity supply from the healthy banks. When liquidity supply is not enough to cover the demand, liquidity crunch happens and the market price of the illiquid asset tumbles. Yet the price tumble will trigger consumers in otherwise healthy banks to withdraw early and force these banks into a run. By "otherwise healthy," we mean that these banks would not have a run had the market price not fallen. It is contagion in the sense that the crisis of some banks spread to others due to liquidity crunch.

We use bank runs to capture the startup of a financial crisis although financial crises can begin with any other sectors. In many industrialized countries, on the one hand, most bank products are now no longer unique and are being provided by an ever growing number of nonbank firms; on the other hand, some of the characteristics of the banking industry culpable for systemic risk also show up in other parts of the financial markets. Many new financial innovations have features that can also generate liquidity problems similar to that of the bank runs. Things like feedback trading, dynamic hedging, and margin calls all have the common feature of selling into a bad market, i.e. sell when price falls, and therefore all have the potential of draining the liquidity of the market. During the fast few years, a host of disturbances, with different degrees of contagion effects, arose in the whole range of financial and derivative markets: The foreign exchange markets experienced the

EMS crises in 1992 and the dollar crises in 1995. The world wide slump in the bond markets in early 1994 came as a total surprise. The futures markets precipitated the *de facto* collapse of Barings and Metallgesellschaft, two venerable and respected companies. The 1987 U.S. stock market crash spilled all over the world. As has been noted by Davis (1994), crises bursting into financial markets have also been shown to exhibit the liquidity problems of the sort encountered in bank runs.

Further, in emerging markets where contagious financial crises happen even more frequently, banks are still playing a very large role in the economies and also in the economic and financial crises. The preeminent role in emerging markets is clarified in Diamond (1997) and stressed in Chang and Velasco (1998). Indeed, as evidenced by the 1994 Mexico crisis, the 1997 Asian crises, and the 1998 Russian crisis, these crises all begin with the banking sector and then spread to the whole economy and also to other countries.

Although the structures of modeling are similar, there are two different views in generating bank runs. One view, which began with Kindleberger (1978) and was developed by Diamond and Dybvig (1983), Bryant (1980), Temzelides (1997), Waldo (1985), and others, argues that bank runs are self-filling prophecies and purely random events ("sun-spots"), unrelated to the real economy. An alternative view, represented by Mitchell (1941), Gorton (1988), and Allen and Gale (1998), argues that financial crises are an inherent part of the business cycle. We take the latter view that financial crises are closely related to the slow down of a fast growing economy. This view confirms with the observation on the recent Asian crises: the economy experienced an apparent slow down before the crises. The empirical study of Gorton (1988) also confirmed the same point. He compares the recessions and panics that occurred in the U.S. during the National Banking Era and has found that the five worst recessions were accompanied by panics. He shows that bank runs were typically preceded by declines in economic indicators such as increased liabilities of failed businesses and declines in stock prices.

Contagion or financial fragility has been investigated from different perspectives. Most recently, Rochet and Tirole (1996) attribute the systemic risk observed in the banking industry to the interbank lending. Lagunoff and Schreft (1997) capture financial fragility by directly modeling the response of an interrelated economy to the exogenous failure of one of the links. Another stream of literature focus on the social learning effect on financial fragility. Examples include, among others, Avery and Zemsky (1995) on multi-dimensional uncertainty and herd behavior in financial markets, and Caplin and Leahy (1994) on endogenous timing. Chamley and Gale (1994) investigates the effects of pure informational externality with both

endogenous timing of decision and endogenous revelation of information. Chari and Kehoe (1997) illustrates with an example on how learning in the sequence of events can generate the herd behavior that drains the liquidity of the market. In all these models, strong-enough externality, though may be generated through different channels, is required to generate the running behavior. Kodres and Pritsker (1999) propose a rational expectations model of securities prices where contagion can arise through correlated information, correlated liquidity shock, and/or cross-market rebalancing. In our model, we use a standard setup of a bank run as a simple mechanism to generate liquidity crunch which, together with imperfect information, brings about the contagion effects.

Studies by Gerlach and Smets (1995), Sachs, Tornell, and Velasco (1996), Valdés (1995), Agénor and Aizenman (1997), and Chang and Velasco (1998) also present explanations of why a crisis in one country might trigger a crisis in another. Masson (1998) tries to distinguish different types of contagion: crises triggered by major economic shifts ("monsoonal effects"), contagion triggered by interdependence among developing countries ("spillovers"), and contagion due to expectation changes("pure contagion"). He explains the "pure contagion" with a balance of payments model that generates multiple ("sunspot") equilibria. Eichengreen, Rose, and Wyplosz (1996), using data for 20 industrial countries from 1959 through 1993, show that the occurrence of crises elsewhere increases the probability of a crisis occurring in a given country, after allowing for the standard set of macroeconomic fundamentals.

The paper is organized as follows. Next section sets up the model. Section 3 analyzes the sources of a bank run and how it becomes contagious. Section 4 explores applications, interpretations and potential extensions of the model. Section 5 speculates on policy implications. Section 6 concludes.

2 The Model

2.1 The Economy

The economy is initially constructed among a continuum of consumers, a finite number N of banks differentiated by their banking efficiency, a consumption good, two assets (one safe asset and one risky asset), and a securities market. An equilibrium is established with a three-period structure. In such an economy, a bank run can start with the most inefficient bank and then, should the aggregate excess supply of

liquid assets be not enough to cover the demand of the running bank, the run may spread to other more efficient banks. In the extreme case where runs spread to the whole banking industry, the market price of the illiquid asset falls to zero, assuming that the technology has no liquidation value.

In a later section, the economy is extended to multiple risky assets to illustrate that contagion in the banking industry can cause co-movement in asset prices even when the underlying technologies are uncorrelated. Further, the economy is extended to infinite periods to illustrate the business cycle effects. The model demonstrates that the economy is more susceptible to contagious bank runs and market crashes after a period of high economic performance when consumers become more willing to invest in a longer horizon and bankers become more optimistic about their risky investments. This optimism from both consumers and investors increases the investment in the illiquid risky assets and decreases the liquidity of the market. As a result, an individual bank run due to the slowdown of the economy is more likely to create the liquidity crunch that initiates the contagion process. On the contrary, the pessimistic and conservative attitudes after a recession or market crash tend to increase the banker's investment in the liquid safe assets. The increased market liquidity makes it harder for any individual bank run to spread to other banks. Inefficient banks are thus more likely to be weeded out of the market without the efficient banks being affected.

2.1.1 Time

There are three periods: $t = 0, 1, 2$. At period 0, consumers are born with an endowment and deposit the endowment in banks. Banks offer deposit contracts to consumers and make investment decisions. At period 1, impatient consumers withdraw their money from the bank and consume; patient consumers make decision on whether to consume or wait till period 2. Banks observe the return to the assets and make portfolio adjustments at this period. The model closes at period 2 when patient consumers, if not having withdrawn at period 1, withdraw their money and consume.

The model can be easily extended to infinite periods: $t = 1, 2, 3, \cdots$, with three subperiods for each period. When these periods are independent, it reduces to the original three-period model. In general, dynamics related to the business cycles can be readily incorporated to the extended time periods by adding correlations between periods.

2.1.2 Consumers

Consumers are born with an endowment at period 0 and will transform the endowment into consumption good in period 1 or 2, depending on whether they are patient or not. Assume that everything they can do, banks can do for them and better, consumers will always put all their money into the bank in period 0.[1] There are two types of consumers: impatient consumers and patient consumers. Let h denote the consumer type: $h = 1$ denotes impatient consumers and $h = 2$ patient consumers. Their utility function can be written as

$$u^h(c_1^h, c_2^h) = g(c_1^h + \beta^h c_2^h), \tag{1}$$

where $\beta^1 = 0$ for impatient consumers and $\beta^2 = 1$ for patient consumers. (c_1^h, c_2^h) is the consumption at period 1 and 2, respectively, for a type h consumers. We further assume that the utility function $g(\cdot)$ is strictly increasing and strictly concave: $g' > 0$ and $g'' < 0$. Also, we assume that $g'(0) = \infty$ such that all consumers have to have some positive consumption.

Let r_2 denote the gross return to an investment from period 1 to period 2. Then the linearity of the indifference curves, implied by the utility function in (1), for both types of consumers says that impatient consumers will choose to consume only at period 1 and patient consumers will choose to consume only at period 2 as long as $0 \leq \beta^1 < 1/r_2 < \beta^2$. As specified later, in our model the return to any investment from period 1 to period 2 is $r_2 = 1$.[2] Therefore, impatient consumers only consume at period 1 with $\beta^1 = 1$. Patient consumers, with $\beta^2 = 1$, are indifferent between period 1 consumption and period 2 consumption. As a result, patient consumers can either imitate the impatient consumer or wait until period 2 to consume, depending on whichever way generates more consumption.

At period 0, consumers are identical and have the same amount of endowment. The preference type (impatient or patient) is revealed in the period 1. Let α denote the fraction of consumers that will turn out to be impatient consumers. It is also assumed to be the probability that a consumer will be of impatient type. As a result, at period 0, each consumer maximizes the expected value of

$$U(c) \equiv \alpha u^1(c_1^1, c_2^2) + (1 - \alpha) u^2(c_1^2, c_2^2).$$

The fraction of impatient consumers α is public information.

[1] Since *a priori* consumers are uncertainty about their types, that is, uncertain about when they need to have the consumption, putting their endowment in the bank and receiving a deposit contract also act as an insurance mechanism, which increases their ex ante expected utility under certain technical conditions, as described in Wallace (1988).

[2] The return can be higher in case of liquidity crunch.

2.1.3 Technology

There is a consumption good and two types of assets: a safe asset and a risky asset. The safe asset can be thought of as a storage technology and has a return of 1. The risky asset is represented by a stochastic production technology that transfers one unit of consumption good at period 0 into R units of consumption good two periods later. We further assume that $\mathbb{E}[R] > 1$, which ensures that even a risk averse investor will always hold a positive amount of the risky asset. $g'(0) = \infty$ ensures that banks have to have some safe investment for impatient consumers.

In a later section, the model will be extended to incorporate multiple risky assets to investigate the spurious correlation between asset price movements even when the underlying production technologies are independent.

2.1.4 The banks

There are N *a priori* identical banks who offer identical deposit contracts to consumers and who make identical investments. Therefore, at period 0, each bank collects an equal amount of endowment E, α of which are from impatient consumers. Bank are differentiated at period 1 by an exogenous random shock to the fixed cost of banking $C_i (i = 1, \cdots, N)$.

In reality, "bad banks" can either have high fixed cost of banking or low return on their investment, or both. The banking cost C_i in this model captures the net effect of investment efficiency.[3] The banking cost shock is public knowledge at period 1.

At period 0, banks know the distribution of the fixed banking cost and make their decisions based on the distribution. Since banks *a priori* are identical, consumers have no preference of choosing one over the other. Each consumer, in general, can be better off by diversifying through all banks such that their consumption will not vary with the exogenous shocks to the banking cost. However, since consumers' decision on consumption is bank-specific, each bank will receive the same amount

[3]It is also possible that some banks have high banking cost but also have high returns, assuming they spend extra money doing research that bears fruits. Or sometimes an imprudent bank may invest in projects which have similar expected returns but much bigger risk. In this model, since the return on the risky asset is the same for all banks, the net difference between banks is captured through the banking cost. It would be equivalent to specify the difference between investment returns. Further, the differentiation of banks in this model is from exogenous shocks. The question as why some banks are less efficient than others is beyond the scope of the paper.

of endowment with the same demographic composition. As a result, whether consumers diversify or not does not affect the banking decision, nor does it affect the potential running behavior or its contagion.

Therefore, we can assume, with no loss of generality, that each bank has a continuum of consumers with a mass of 1 and an aggregate endowment of E. α fraction of the endowment is from impatient consumers.

The Bertrand type competition between banks drives their profits to zero and forces banks to offer a contract that maximizes the expected utility of their customers. Green (1987) and Green and Oh(1991a, b) obtain similar results under related contexts. An informal proof goes as follows: Suppose there are two banks competing with each other to offer the deposit contracts. A bank who offers no contract earns zero profit. If one bank offers a contract that does not maximize the consumer's expected utility, then the other bank can always bid away all the consumers by offering a contract that slightly increases the consumer's expected utility. As this process continues, banks will converge to offering a contract that maximizes the consumers' expected utility and yields zero profits for the banks.

Similar arguments can be extended straightforwardly to a profit-maximizing monopolistic bank, who would extract rents from the consumers but would still offer them the same kind of expected-utility maximizing contracts: Only by providing a deposit contract that maximizes consumers' expected utility can the banks extend the consumers' demand curve to the maximum and thus can they extract the maximum rent from the consumers.

We also assume that consumers observe banks' investment decision so that no moral hazard problems arise regarding the investment. Banks not only offer the same optimal deposit contract but also make the same investment decision that maximizes consumers' expected utility.[4]

At period 1, knowing the banking cost, realized return, and their liquidity demand, banks make portfolio adjustments in the securities market.

[4]If consumers do not observe banks' investment decision and the banking cost is endogenously determined by the efforts they put into the investment decision, moral hazard problems may arise and banks may differ in their investment decisions (unobservable) while offering the same deposit contract (observable).

2.1.5 The securities market

The securities market is formed among the bankers to rebalance their portfolio after observing the return to the risky investment. Since for now we assume that there is just one risky asset, and everybody observes the same information, the primary role of the security market is to reallocate liquidity between different banks. No trade occurs in absence of bankruptcy since each bank has sufficient liquidity (safe assets) to fulfill their obligation. The market price of the risky asset will reflect its fundamental return in period 2. Normalize the price at period 0 to 1: $P_0 = 1$, we would have the price for the risky asset at period 1 equal to the true return: $P(R) = R$. On the other hand, when some banks do not have enough safe assets to fulfill their commitment to early withdrawals, they are forced to exchange their risky asset holdings for the safe (liquid) assets. The price of the risky asset, $P(R)$ will then be determined by the real return of the asset and the liquidity constraint.

The real return R are known to all bankers, based on their research on the technology; however, consumers only observe the market price of the asset and base their decision on the market price. The contagion behavior depends crucially on this assumption of imperfect information.

2.2 The deposit contract

We define a *standard deposit contract* to be one that promises a fixed amount at period 1 at normal times and pays out all available liquidable assets, divided equally among those withdrawing, in the event that the bank does not have enough liquid assets to make the promised payment. Let $\overline{c}$ denote the fixed payment promised to early consumers: consumers withdrawing at period 1. We can ignore the payment promised to the late consumers since they are always paid whatever is left at the last date.

At time 0, each bank receives an identical amount of endowment E and invest the endowment between the safe asset and the risky asset. Let L and X denote the fraction of fund invested in the safe asset and the risky asset, respectively, with $L + X = E$. Each bank chooses a portfolio (L, X) and a promised payout $\overline{c}$ at period 0 to maximize the expected utility of their clients. Since all banks have the same amount of endowment and face the same information set, they make identical decisions. The standard deposit contract requires each bank to pay $\overline{c}$ to an early consumer at date 1, and to liquidate all its assets otherwise. If there is no bank run,

the early consumers receive $c_1(R) = \overline{c}$ each and late consumers receive whatever is left:

$$c_2(R) = \frac{L - \alpha\overline{c} - C_i + RX}{1 - \alpha}, \tag{2}$$

where C_i is the fixed cost of bank i. When there is a bank run, all consumers split the liquidated assets of the bank,

$$c_1(R) = c_2(R) = L + P(R)X - C_i,$$

where $P(R)$ is the market liquidating price of the risky asset, which, as demonstrated later, is a function of the true return R and the liquidity constraint of the securities market.

At period 1, impatient consumers always withdraw. Patient consumers will make comparison between the benefits of withdrawing in period 1 and waiting until period 2 to see whether it is worth waiting, based on their observations of the fixed cost, C_i, of their associated banks and the market price of the risky asset, $P(R)$. The patient consumers will wait if and only if the expected utility of waiting is higher than consuming in period 1. However, due to the introduction of the securities market and portfolio rebalancing at period 1, the payoff at period 2 is not exactly the same as shown in equation (2), but should be

$$c_2(R) = \frac{\frac{R}{P}(L - \alpha\overline{c} - C_i) + RX}{1 - \alpha}.$$

It captures the effect that if the banks can buy the risky project at a discount at period 1 using its residual cash, it can actually generate a higher consumption package for the later consumers. Patient consumers wait if and only if

$$\mathbb{E}[g(c_2(R))|P] \geq g(\overline{c}), \tag{3}$$

where $\mathbb{E}[\cdot|P]$ captures the patient consumers' expectation conditional on the observation of the market price. For simplicity, we assume that consumers regard the market price $P(R)$ as the true return to them. Then condition (3) is reduced to:

$$\frac{L - \alpha\overline{c} - C_i) + P(R)}{1 - \alpha} > \overline{c}, \tag{4}$$

which comes to the critical condition:

$$P(R) > \frac{\overline{c} - L + C_i}{X} = R_i^*, \tag{5}$$

where R_i^* is defined as the *threshold value* for bank i to have a run. *Observing the banking cost and the market price of the risky asset, the patient consumer will wait*

if and only if the market price of the risky asset is higher than the threshold value of his or her bank. When the market price $P(R)$ of the risky asset is lower than the threshold value of a bank, all consumers in this bank will decide to withdraw at period 1, thus incurring a bank run.

The assumption that consumers regard the market price as the true return greatly simplifies the analysis but seems hard to swallow because in equilibrium the market price $P(R)$ can be either equal to R when there is enough liquidity in the market or lower than R when there is a liquidity shortage. So naturally the expected return conditional on the market price $\mathbb{E}[R|P]$ should be higher than the price P and therefore the patient consumers should have a better incentive to wait. However, in Section 3 we will show that under our specific assumptions regarding the distributions of R and C_i, it is indeed an equilibrium that the actual return to an infinitesimal consumer is equal to the market price. In Section 4, we explore conditions when this is not an equilibrium and conditions when there is no contagion at all.

In this model, we assume that all consumers agree on (5) such that the equilibrium is unique: The run happens for a bank if and only the market price is below its threshold value: $P(R) < R_i^*$. With strong enough perturbation to this assumption, multiple equilibria may result with bank run always being one of them. Suppose for some reason (say, some exogenous shock) some patient consumers become less patient ($\beta^2 < 1$), or they come to believe that the return to their bank's risky investment is lower than the threshold value, contrary to what is shown by the market price, or they believe that the fixed cost of their bank is actually bigger than what is commonly believed. In any case, if they decide to withdraw early regardless of the critical condition (5) to the point that the bank does not have enough safe assets to pay them $\overline{c}$, then other patient consumers may also be better off by withdrawing early and split the pie together with them because there may be less than $\overline{c}$ or even zero left for period 2 consumption. This is basically the "sunspot" view of banking panics, expressed in Diamond and Dybvig (1983).

An underlying assumption of Diamond and Dybvig (1983) is the *sequential service constraint* at period 1: Consumers withdraw at a random order and the bank does not know how many consumers are going to withdraw until the last minute of the period. Wallace (1988) provides a formalization of this sequential-service constraint and proves that, when this constraint being taken seriously, the deposit insurance (suspension after α withdrawals) mechanism proposed in Diamond and Dybvig (1983) is not really feasible. By assuming that it is common knowledge that all consumers agree on the critical condition (5) and the parameter values in the

condition (5), our model does not have a big role for the sequential-service constraint. Since by observing the market price and its banking cost, the bank knows with certainty whether it will have a run or not. The arrangement of splitting all its liquidated assets in case of a run and paying $\overline{c}$ in case of no run is feasible regardless of whether consumers come in sequence or not.

Green and Lin (1996) proposes a contract that can potentially kill all bank runs. Under such a contract, the promised consumption to either early or late consumers is not fixed, but rather state-contingent, such that at any state the promised consumption for early consumers is always slightly lower than the promised consumption for later consumers. As a result, patient consumers will have no incentive imitating impatient consumers and will consume at period 2 in all states. The state space in our setup can be captured by a triple that includes the banking cost, the realized return of the risky investment, and the sequence of reporting for the consumer (if sequential service is observed). Such an arrangement is ex post efficient and has no runs in equilibrium. However, any such state-contingent arrangement (contract) will leave room for moral hazard problems from the bank's side: The bank may *ex post* misrepresent itself by claiming a lower return to the risky asset, a higher cost of its banking practice, or a larger number of consumers that have withdrawn money from the bank so that the bank pays the consumer less amount of the consumption good than it ought to.[5] Furthermore, anticipating this possibility and its consequences that less consumption will be available at period 2, patient consumers may be tempted to withdraw early, which would increase the likelihood of a bank run. This may be why such contracts are rarely seen in practice. In essence, contracts that depend on unverifiable states are not feasible in practice.

At period 0, the bank's decision problem can be summarized as follows

$$\begin{array}{lll} \max_{\{\overline{c},(L,X)\}} & & \mathbb{E}\left[\alpha g(c_1(R)+(1-\alpha)g(c_2(R))\right] \\ \text{s.t.} & \text{(i)} & L+X\geq E; \\ & \text{(ii)} & c_1(R)=\begin{cases} \overline{c} & \text{if} \quad P(R)\geq R_i^* \\ L+P(R)X-C_i & \text{if} \quad P(R)<R_i^* \end{cases}; \\ & \text{(iii)} & c_2(R)=\begin{cases} (L-\alpha\overline{c}+RX-C_i)/(1-\alpha) & \text{if} \quad P(R)\geq R_i^* \\ L+P(R)X-C_i & \text{if} \quad P(R)<R_i^* \end{cases}. \end{array} \tag{6}$$

The expectation operator $\mathbb{E}[\cdot]$ is taken over the distribution of the banking cost C_i and the return R on the risky asset. The bank offers the standard deposit

[5] In the extreme, the bank could claim that it is hit by a huge banking cost shock and nothing is left for any of its consumers. Unless all relevant state information is verifiable with no or little cost, misrepresentation is likely to happen.

contract $\overline{c}$ and makes portfolio decisions (L, X) to maximize the expected utility of its consumers. Condition (i) says that the investments (in the safe and risky assets) cannot be more than the endowment. Condition (ii) says that the impatient consumers obtain $\overline{c}$ as promised if there is not a bank run ($P(R) \geq R_i^*$) or split with patient consumers the liquidated wealth when there is a run. Condition (iii) specifies the payoff structure for patient consumers: they obtain the residual of the wealth when there is not a bank run and this residual payment is greater than the promised payment to the impatient consumer, $\overline{c}$. When there is a run, patient consumers withdraw together with impatient consumers at period 1 and split the liquidated assets.

2.3 The market prices

The safe asset can be regarded as cash. It always has a price of 1 per unit. We normalize the market price of the risky asset at period 0 to 1. Then at period 2, the production matures and it generates a return of R, so the market price is R at period 2. At period 1, the market price of the risky asset $P(R)$ depends both on the return to the production technology at period 2 and on the liquidity constraint at period 1.

At period 1, recognizing the real return of the technology at period 2, bankers will charge a price for the risky asset that is equal to its real return: $P(R) = R$, when the market have enough liquidity to sustain banks' operation.

When there are bank runs, the running banks are forced to liquidate their holdings of the risky asset. Other banks can absorb the excess supply of the risky assets with their excess stock of safe assets, the residual safe asset after paying off early consumers.

Let k denote the number of bankrupt banks. Let $L_s = \sum_{i=1}^{N-k} \max(L - \overline{c} - C_i, 0)$ be the aggregate excess supply of the safe assets from the $(N-k)$ healthy banks. Let R^0 be implicitly defined by the condition

$$L_s = R^0(kX).$$

Assume that the banks are sorted by its banking cost: $C_1 > C_2 > \cdots > C_N$, that is, bank 1 is the least efficient bank and bank N is the most efficient one. Then the market price of the risky asset is governed by

$$\begin{cases} \text{(i)} & P(R) = R & \text{if} \quad R < R^0 \text{ or } R \geq R_1^*; \\ \text{(ii)} & P(R) = L_s/(kX) & \text{if} \quad R^0 < R < R_k^*. \end{cases} \tag{7}$$

In other words, the price will be forced below its true value only if the return is low enough to provoke a run but not so low that the market is liquid enough to absorb the asset at its "fair" value.

In the model, once the patient consumers decide to withdraw their money from the bank based on their observation of the market price and the banking cost, they will withdraw and consume the money instead of reinvesting the money in another more efficient bank. This obviously aggravates the liquidity problem. Based on the utility function specification in (1), patient consumers do not have any incentive to reinvest. Patient consumers from the running banks can only reinvest their money in the surviving banks. However, a surviving bank, knowing the true return to the technology, has no incentive to sell the risky asset back to a consumer at any price lower than the true return if a patient consumer wants to buy the risky asset from the bank. Following the same logic, the surviving bank does not have any incentive to offer him any consumption higher than what he deposits, either, if the patient consumer wants to deposit his or her money in the bank at period 1. Knowing so about the bank, the consumer can only expect a maximum return of 1 for reinvestment. With a return of 1, a patient consumer is indifferent between consuming now and waiting for another period. If the bank tries to assign part of the banking cost to the reinvesting consumer, the consumer will definitely be better off consuming at period 1 than reinvesting.[6]

2.4 The Equilibrium

We have defined the bank's optimal contract decision and their portfolio decision in (6) and the securities market clearing condition in (7). An equilibrium of the economy is defined as follows

Equilibrium 1 *An equilibrium for the economy consists of a price function* $(P(R))$ *that clears the securities market as described in (7), given the deposit contract* $((L, X), \overline{c})$*, and a deposit contract* $((L, X), \overline{c})$ *that maximizes the consumers' expected utility as described in (6), given* $P(R)$*.*

[6]In case of severe liquidity crunch, a healthy bank may have an incentive to offer a one-period return that is higher than 1 to attract extra liquidity. With the extra liquidity, the bank can buy up more risky asset at a discount price and can therefore increase the consumption of the later consumers. However, since consumers do not observe the true return and only observe the market price, any offers with a return higher than 1 at period 1 will be subject to the opposition of the bank's old customers.

3 Characterization of the Equilibrium

3.1 Sources of a bank run

As we argued earlier, a bank run occurs if and only if the market price of the risky asset is lower than the threshold value R_i^*:

$$R_i^* = (\overline{c} - L + C_i)/X \tag{8}$$

Obviously, *a low return to the investment* is one of the most direct sources of a bank run. A low return to the risky investment captures the effect of the shocks to the *real economy.* A bank run is a direct result of a slowdown of, or a negative shock to, the economy.

A bank run can also be caused by *inefficient banking practice*, as captured by the banking cost C_i in this model. Given the optimal contract and the optimal portfolio decision: $((L, X), \overline{c})$, which are identical among all banks, the threshold value for a bank run increases with the banking cost C_i. Therefore, under the same economic condition, an inefficient bank (with high banking cost) has a high threshold value and is hence more difficult to survive. In reality, the quality of a bank or banking practice can be captured either by their cost of banking or by their ability of picking the right investment opportunities, or both. In this model, we normalize the investment return to be the same across banks. The quality difference is solely captured by the banking cost. However, since shocks to the banking cost is exogenous, the model does not answer questions as why banks have different efficiencies.

The setup is greatly simplified by the assumption that at period 0 banks make contract designs and portfolio decisions without knowing their own efficiency of banking. If consumers *a priori* know the efficiency of each bank, they will all deposit their money to the most efficient bank. If banks *a priori* know their banking costs as private information, they will try to hide the information unless they are the most efficient one. The most efficient bank, knowing that it is the best, will try to signal to consumers that he is the best by offering a deposit contract that maximizes consumers' expected utility given the least cost. All the other banks then have to offer the same contract to disguise themselves. If investment decision is observable to consumers, all banks have to make the same investment decision as the most efficient does. As a result, the less efficient banks will have a higher probability of having runs because, assuming a smaller cost, they have to offer to pay early consumers a higher consumption $\overline{c}$ than they can really afford based on their real

banking cost. If the portfolio decision is unobservable to consumers, then bankers, as long as they are less risk averse than consumers, will have a tendency to make over-risky investments to increase their chance of survival.

In addition to banking cost and realized return to the risky investment, the *demographic composition* of consumers also affect the possibility of a bank run indirectly through its effect on the banking decision. Consumer demographics is captured by α, the fraction of impatient consumers. The banking decision, $((L, X), \overline{c})$, is a function of α. When there are more impatient consumers, the banks are forced to invest more in the liquid safe assets.[7] As a result, the expected wealth will be smaller since the safe asset has a lower return than the risky asset: $E(R) > 1$. Banks will then have to offer a smaller consumption bundle $\overline{c}$ to the early consumers. In summary, an increase in α (impatient consumers), will increase the safe asset investment L and decrease the risky asset investment X as well as the promised payment to early consumers $\overline{c}$. Its effect on the threshold value, as given in (8), can be either way, depending on the relative change in $\overline{c}$, L, and X. However, as will be proved immediately, an increase in impatient consumers (α) will make the threshold value more sensitive to banking efficiency (C_i) while an increase in patient consumers blurs the difference between banks with different banking cost.

To summarize, we propose

Proposition 1 *(1) A bank run can either be a result of business cycles, as captured by the slowdown of the economy (low realized returns for the risky asset), or be a result of inefficient banking practice, as captured by high banking cost, or both. (2) Demographic composition affects bank runs through its effects on the banking decision. When there are more patient consumers, banks invest more in the risky asset and offers more to the early consumers. More importantly, as a result of these banking decisions, the threshold values for bank runs become closer to each other for banks with different efficiencies.*

Proof: The first part of the proposition is self-obvious from the previous analysis and from (8). The second part can be proved easily by finding the partial derivative of the threshold value R_i^* on the banking cost C_i:

$$\frac{\partial R_i^*}{\partial C_i} = \frac{1}{X}.$$

[7] In the extreme, if all consumers are impatient, the banks have to invest all the endowment in the safe asset. When all consumers are patient, however, there will still be some investment in the safe asset due to the risk aversion of the consumers.

Since the risky investment X increases with patient consumers, the slope of the threshold value versus banking becomes flatter with more patient consumers. ■

Intuitively, with more patient consumers, all the banks will invest more in the risky asset and thus have a higher expected revenue. The fixed banking cost C_i becomes relatively small compared to the total revenue of these investments. Thus the threshold values for different banks become relatively insensitive to the difference of their banking cost.

3.2 Three phases of an economy: A numerical example

Depending on the realized return to the risky technology and the realized distribution of banking efficiencies across banks, an economy can be totally healthy without a single crisis, or it can have idiosyncratic crisis which is not contagious. Of course, the economy can also have contagious bank runs and securities market crashes. These three phases of the economy can be best illustrated by the following numerical example.

To be exact, we assume log utility: $g(c) = \ln(c)$. We also assume that the return to the risky investment is drawn from a uniform distribution $f(R)$ defined on $[\underline{R}, \overline{R}]$:

$$f(R) = \begin{cases} 1/(\overline{R} - \underline{R}) & \text{for } \underline{R} \leq R \leq \overline{R}; \\ 0 & \text{otherwise.} \end{cases}$$

The mean return is $\mathbb{E}[R] = (\underline{R} + \overline{R})/2$. Also assume that the banking cost is uniformly distributed between $[\underline{C}, \overline{C}]$:

$$g(C_i) = \begin{cases} 1/(\overline{C} - \underline{C}) & \text{for } \underline{C} \leq C \leq \overline{C}; \\ 0 & \text{otherwise.} \end{cases}$$

The mean cost is then $(\underline{C} + \overline{C})/2$.

For ease of solving the problem, we assume that investors do not expect bank runs *a priori*.[8] We can solve for the optimal contract when there is no bank run. With $c_1(R) = \overline{c}$, $c_2(R) = (L+RX-\alpha\overline{c}-C_i)/(1-\alpha) = [(R-1)X-\alpha\overline{c}+E-C_i]/(1-\alpha)$, the maximization in (6) can be reduced to an unconstrained problem:

$$\max_{\overline{c},X} \ \mathbb{E}\left[\alpha \ln(\overline{c}) + (1-\alpha)\ln\left(\frac{(R-1)X - \alpha\overline{c} + E - C_i}{1-\alpha}\right)\right].$$

[8]Cases when banks ex ante expect bank runs and contagion are considered in Section 4.5. In general, such knowledge about bank runs makes banks' decision more conservative. See Proposition 4.

The two first order conditions are

$$\begin{cases} \mathbb{E}\left[(\overline{c})^{-1} - (1-\alpha)\left((R-1)X - \alpha\overline{c} + E - C_i\right)^{-1}\right] &= 0; \\ \mathbb{E}\left[\left((R-1)X - \alpha\overline{c} + E - C_i\right)^{-1}(R-1)\right] &= 0; \end{cases}$$

from which the optimal contract $\overline{c}$ and the portfolio decision X can be solved. L is obtained by $L = E - X$. Integration over R and C_i yields

$$\begin{cases} \left[X\left(\overline{R}-\underline{R}\right)\left(\overline{C}-\underline{C}\right)\right] / \left[(1-\alpha)\,\overline{c}\right] = \\ -\left(X\left(\overline{R}-1\right)+E-\alpha\overline{c}-\overline{C}\right)\ln\left(X\left(\overline{R}-1\right)+E-\alpha\overline{c}-\overline{C}\right) \\ +\left(X\left(\overline{R}-1\right)+E-\alpha\overline{c}-\underline{C}\right)\ln\left(X\left(\overline{R}-1\right)+E-\alpha\overline{c}-\underline{C}\right) \\ +\left(X\left(\underline{R}-1\right)+E-\alpha\overline{c}-\overline{C}\right)\ln\left(X\left(\underline{R}-1\right)+E-\alpha\overline{c}-\overline{C}\right) \\ -\left(X\left(\underline{R}-1\right)+E-\alpha\overline{c}-\underline{C}\right)\ln\left(X\left(\underline{R}-1\right)+E-\alpha\overline{c}-\underline{C}\right); \\ \\ X\left(\overline{R}-\underline{R}\right)\left(\overline{C}-\underline{C}\right) = \\ \left[X^2\left(\overline{R}-1\right)^2-\left(E-\alpha\overline{c}-\overline{C}\right)^2\right]\ln\left(X\left(\overline{R}-1\right)+E-\alpha\overline{c}-\overline{C}\right) \\ -\left[X^2\left(\overline{R}-1\right)^2+\left(E-\alpha\overline{c}-\underline{C}\right)^2\right]\ln\left(X\left(\overline{R}-1\right)+E-\alpha\overline{c}-\underline{C}\right) \\ -\left[X^2\left(\underline{R}-1\right)^2+\left(E-\alpha\overline{c}-\overline{C}\right)^2\right]\ln\left(X\left(\underline{R}-1\right)+E-\alpha\overline{c}-\overline{C}\right) \\ +\left[X^2\left(\underline{R}-1\right)^2-\left(E-\alpha\overline{c}-\underline{C}\right)^2\right]\ln\left(X\left(\underline{R}-1\right)+E-\alpha\overline{c}-\underline{C}\right). \end{cases} \tag{9}$$

Given parameters on the endowment E and on the demographic composition α, as well as the range for the risky asset return $[\underline{R}, \overline{R}]$ and the range for the banking cost $[\underline{C}, \overline{C}]$, we can solve for X and $\overline{c}$.

As an example, we set $E = 10$, $\alpha = 0.50$, $[\underline{R}, \overline{R}] = [0, 3]$, and $[\underline{C}, \overline{C}] = [0, 1]$. The expected return is then greater than 1: $\mathbb{E}[R] = 1.5 > 1$. The solution for the optimal contract is $\overline{c} = 9.47$. The optimal portfolio allocation are $(L, X) = (6.63, 3.37)$.[9] By varying the value for α, we can compute the banking decisions and compare the

[9] As a diversion, suppose that banking qualities are private information and each bank is trying to pretend that it is the most efficient bank by mimicking the most efficient bank's decision. For the most efficient bank, the banking cost is the lower bound of the distribution, which is 0. Then the integration will yield directly the optimal contract: $\overline{c} = E - C_i = 10$ ($C_i = 0$). X is solved from the following equation

$$\frac{X\left(\overline{R}-\underline{R}\right)}{(1-\alpha)\,E} = \ln\left[\frac{X\left(\overline{R}-1\right)+(1-\alpha)}{X\left(\underline{R}-1\right)+(1-\alpha)}\right].$$

We obtain the optimal solution for X at $X = 3.58$. L is then solved by $L = E - X = 6.42$. Compare this solution with the results in the text where banking cost is uniformly distributed between $[0, 1]$, we see that all banks (except the most efficient one) are offering a higher consumption $\overline{c}$ than they can afford to and they also invest more in the risky asset than they should have.

effects of the demographic composition on the banking decisions $(L, X, \overline{c})$. Figure 2 illustrates the optimal contract $\overline{c}$ and the optimal allocation to the risky asset X as a function of the demographic composition α. As expected, both are decreasing with the increase of impatient consumers.

Without impatient consumers, the optimization is reduced to a standard capital allocation problem which optimizes under the trade off of risk and return. The incorporation of impatient consumers constitutes a liquidity constraint to the optimization problem. The more impatient consumers the bank has, the more rigid the liquidity constraint is, the more the bank has to invest in the safe liquid asset.

Given the solution to the optimal contract $\overline{c}$ and the optimal allocation (L, X), the threshold value of return can be solved as a function of the banking cost and demographic composition based on (8). Figure 3 illustrates the effects of demographic composition on the threshold values for different banks. As expected from Proposition 1, the slope of the curve decreases, i.e., the difference between banks are reduced, as the fraction of impatient consumers decreases.

Following the same example, we now illustrate how, depending on where the realized return to the risky asset lies, the banking industry can (1) be totally healthy, (2) have individual bank runs with no contagion, and (3) have bank panics, where crises spread to the whole banking industry and the securities market.

Recall the previous example, with $\alpha = 0.50$, the optimal contract offered by the banks are $\overline{c} = 9.47$. The portfolio decisions are $(X, L) = (3.37, 6.63)$. We further assume that there are 6 banks $(N = 6)$ with uniformly distributed fixed costs: $C_1 = 1; C_2 = 0.8; ...; C_6 = 0$.[10] The threshold values for these banks can then be obtained from (8) as

$$R_i^* = (\overline{c} - L + C_i)/X = [1.14; 1.08; 1.02; 0.96; 0.90; 0.84].$$

Depending on where the realized return lies, we can either have no bank runs, no contagion, or contagion. Obviously, to have a bank run, the realized return has to be below the threshold value of the least efficient bank. To have contagion, the liquidity of the market has to be insufficient to absorb the liquidity needs: liquidity crunch has to happen.

[10] Alternatively, we can assume that there is a continuum of banks with banking cost uniformly distributed between $[0, 1]$. The result will change little except that bank panics happen now when a certain fraction of banks goes bankrupt. A finite number of banks fits the reality better.

3.2.1 Healthy economy

When the return R of the risky investment is higher than 1.14, the threshold value of the worst bank, no bank has troubles. The whole banking industry is healthy. Impatient consumers of all banks obtain what they are promised: $c_1(R) = \overline{c} = 9.47$. Patient consumers wait till period 2 to consume the residual

$$c_2(R) = \frac{L - \alpha\overline{c} + RX - C_i}{1 - \alpha} \geq \overline{c},$$

which is greater than the consumption of impatient consumers but is different for consumers from different banks: patient consumers in the more efficient banks (smaller banking cost) consume more than consumers in inefficient banks.

3.2.2 Idiosyncratic bank run

When the return to the risky asset is lower than the threshold value of the least efficient bank but is higher than the second worst: $1.14 < R < 1.08$, the worst bank (bank 1) will have a run. Consumers of bank 1, patient or impatient, will together split the liquidated asset of the bank

$$c_1(R) = c_2(R) = L + P(R)X - C_1.$$

The price of the risky investment, $P(R)$, as in (7), depends both on the return R and on the liquidity of the market. The liquidity supply is measured by the aggregate excess supply of the safe asset, L_s. The excess supply of safe assets for the six banks, in case of no bankruptcy, are, respectively

$$L_i = \max(L - \alpha\overline{c} - C_i, 0) = [0.89; 1.09; ...; 1.89].$$

Assume that bank 1 is indeed the only one that runs ($k = 1$), then the aggregate excess supply of the safe asset is:

$$L_s = \sum_{i=2}^{6} L_i = 7.45$$

The liquidity demand from bank 1 is $X = 3.37$, which is smaller than the supply. The critical value of the asset market is therefore which is higher than the realized return $1.14 < R < 1.08$:

$$R^0 = \frac{L_s}{kX} = 2.21, \tag{10}$$

This means that there is enough liquidity (safe asset) to absorb the risky asset from bank 1 (the least efficient bank). No liquidity crunch occurs. The market price of this asset equals the real return: $P(R) = R$. Bank 1 is the only bank that goes bankrupt. There is no contagion on other banks.

For return $R \in [1.08, 1.14]$, the consumption of the consumers in bank 1 is between

$$c_1(R) = c_2(R) = L + RX - C_1 \in [9.27, 9.47].$$

Since no other bank follows the suit, this is the period when only individual bank runs with no contagious effect. This happens when the running banks only account for a small fraction of the banking industry and the market has enough liquidity supply such that the liquidity need of the falling banks can be totally absorbed by other healthy banks.

3.2.3 Contagious bank run

Suppose now the realized return to the risky asset is $R = R_2^* = 1.08$, then at least two of the six banks have runs. Suppose indeed only these two banks run, similar calculations give us the liquidity supply and demand, as well as the critical value for the security market:

$$L_s = \sum_{i=3}^{6} = 6.36, \qquad L_d = 2X = 6.74, \qquad R^0 = 0.94.$$

Note that the liquidity demand (L_d) is now higher than the supply and the critical value R^0 is lower than the real return $R = 1.08$, which implies that there is a liquidity crunch and that the market price of the risky asset will fall to $R^0 = 0.94$: $P(R) = 0.94$. Yet, this price is even lower than the threshold value of the fourth bank $R_4^* = 0.96$: Bank runs are now inevitable for the third and fourth banks also. This additional bank run in turn means that we have a bigger liquidity need and a smaller liquidity supply. With four banks on the run and two banks left providing potential liquidity, the aggregate liquidity supply becomes $L_s = 3.58$. The liquidity demand is $4X = 13.49$. Market clearing yields the new market price of the risky asset:

$$P(R) = L_s/(4X) = 0.26,$$

which is even lower than threshold value of the best bank ($R_6^* = 0.84$). As a result, the whole banking industry tumbles. A widespread bank panic occurs. Since the risky technology cannot be liquidated at period 1 (liquidation value is zero), and

there is no liquid asset in the market to absorb the risky asset, the market price of this risky asset falls to zero.

Figure 4 illustrates the consumption of the consumers of the least efficient bank (bank 1) as a function of the risky asset return. As illustrated above, the economy can be divided into three phases. In phase I, where $R \geq R_1^* = 1.14$, no bank goes bankrupt. Early consumers consume $\overline{c} = 9.47$ as promised while late consumers consume the residual, which is greater than $\overline{c}$ and increases with return R. In phase II, $1.08 = R_2^* < R < R_1^* = 1.14$, the least efficient bank runs, but its risky asset can be liquidated at its true value. Both patient and impatient consumers receive the same consumption at period 1: $c_1(R) = c_2(R) = L + RX - C_1$. In phase III, when the return on the risky asset $R \leq R_2^*$, a contagious bank run occurs: all banks run together. The risky asset cannot be liquidated and thus has zero value. Both impatient and patient consumers split the safe asset: $c_1(R) = c_2(R) = L - C_1$.

Proposition 2 *Depending on the status of the economy (the realized return to the risky asset) and the distribution of bank threshold values, the banking industry can be totally healthy, experiencing some idiosyncratic bank runs with no contagion effects, or experiencing a contagious bank panics and securities market crashed.*

Figure 5a depicts the equilibrium price as a function of the true return to the risky project. Under the uniform distribution assumptions on both return R and the banking cost C_i, there is either no contagion or complete contagion, and the equilibrium price of the risky project is either equal to the true return or falls down to zero:

$$P(R) = \begin{cases} R & when \quad R > R^{**} \\ 0 & when \quad R \leq R^{**} \end{cases}$$

Under the assumptions specified in the numerical example, the critical return threshold for complete contagion to happen is $R^{**} = 1.08$.

Recall that this equilibrium is obtained under the assumption the consumers regard the market price as the actual return to them. This is indeed an equilibrium under our set-up: When $R > R^{**}$, the equilibrium market price is indeed equal to the true return; when $R \leq R^{**}$, a complete contagious panics occurs and the decision of an infinitesimal consumer will not be able to reverse that. As a result, regardless of the true return, the consumer can only run with the mass and end up with zero return on the risky project. Since under such a complete contagion scenario, all banks have to be liquidated at period 1, residual consumption at period 2 is zero, it is optimal for all infinitesimal consumers to withdraw at period 1.

Remark 1 *Under the parameter specification of the numerical example, the economy either has no contagion or complete contagion. The equilibrium market price for the risky project either reflects the true return or equals zero. It is consistent with the equilibrium for the consumers to regard the market price as the actual return to them.*

In reality, however, most of the financial crises are contagious only to a certain degree. Complete contagion where the whole world economy breaks down is rare if any. Under our current set up, one way to generate this is to assume that the banking cost, instead of uniformly distributed, has a block-wise distribution. Now let us assume that *ex ante* the banking cost is still uniformly distributed between $[0, 1]$ so that we can use most of the results from the above numerical example. The only think we intend to change is the ex post realization of the banking cost. To increase some maneuvering room, we increase the number of banks to 22 while still assuming that each bank receives an endowment of $E = 10$. Each bank's decision is therefore the same as before: $\overline{c} = 9.47, (X, L) = (3.37, 6.63)$. Assume that the ex post realization of the banking cost falls into three clusters. The first group has 7 banks, all with a banking cost of 1, the second group has one bank with a banking cost of 0.9, the last group has 14 banks, all with a cost of 0.13:

$$C_i = \{1, \cdots, 0.9, 0.13, \cdots\}.$$

The corresponding threshold value is therefore

$$R_i^* = \{1.14, \cdots, 1.11, 0.88, \cdots\}.$$

Again, when the realized return is greater than $R_1 = 1.14$, we have a health economy where no bank runs and market price reflects the true return. However, when the realized return falls below $R_1 = 1.14$, say $R = 1.13$, then at least seven banks will be forced to be liquidated. The liquidation need is $7 \times X = 23.61$ and the liquidity supply $L_s = \sum_{i=8}^{22} L_i = 25.63$. The critical value is therefore

$$R^0 = \frac{L_s}{7X} = 1.08,$$

which is lower than the true return $R = 1.13$, implying a liquidity crunch. The market price of the risky project therefore falls to $P(R) = R^0 = 1.08$. Yet, this price is lower than the threshold value of the second group of bank ($R_i^* = 1.11$), leading to a contagious run to this group. The liquidation need now increases to 8 banks: $8 \times X = 26.98$ while the liquidity supply decreases to those from the last 14 banks: $L_s = \sum_{i=9}^{22} L_i = 24.64$. The critical value becomes

$$R^0 = \frac{L_s}{8X} = 0.91.$$

Now the even tighter liquidity drives the market price further down to $P(R) = 0.91$. This price, however, is not enough to triggers runs in the third group of banks because they have a threshold value of 0.88. There is contagion, but the contagion is partial. Only when the realized return is lower than 0.88 will the whole market totally collapse. The market price function, as in depicted in Figure 5b, can be written as

$$P(R) = \begin{cases} R & when \quad 1.14 < R \leq 3; \\ 0.91 & when \quad 0.91 < R \leq 1.14; \\ R & when \quad 0.88 < R \leq 0.91; \\ 0 & when \quad 0 < R \leq 0.88. \end{cases} \tag{11}$$

However, this price function is obtained with the assumption that consumers regard price as the actual return to them on the risky project. Obviously, this assumption is violated when the realized return falls the range $R \in (0.91, 1.14]$. Now we can check whether knowing the price function in (11) will change the consumers' decision. Since the return return is fully revealed when $R > 1.14$ and the decision becomes trivial when $R \leq 0.88$. The only consumers who may change their decision are the patient consumers in the second group of bank ($C_i = 0.9$) when the market price is 0.91. Conditional on $P = 0.91$, the actual return is not revealed but consumers know that the true return is uniformly distributed between 0.91 and 1.14. The expected utility of consuming at period 2 is,

$$\mathbb{E}[g(c_2(R))|P = 0.91] = \int_{0.91}^{1.14} \ln\left[\frac{(R/P)(L - \alpha\overline{c} - C_i) + RX}{(1-\alpha)}\right](1.14 - 0.91)dR = 2.21.$$

Comparing this with the maximum utility of consuming at period 1

$$g(\overline{c}) = \ln[\overline{c}] = 2.25,$$

the patient consumers at the second group of bank will not change their decision of withdrawing early (running). Therefore, the price function in (11) represents an equilibrium under the current set-up.

This modified example, of course, remains very stylized and even naive compared with the real situation of the economy. Yet it presents a clean picture of how liquidity crunch can arise in an economy and how such an liquidity crunch, coupled imperfect information about the underlying investment, can lead to contagion in financial markets. In the next subsection, we investigate under what circumstances contagion is more likely to happen.

3.3 Sources of contagion

From the above analysis, we can see that the existence and degree of contagion is directly related to the realized state of the economy, as captured by return to the risky asset. When the realized return is higher than the threshold value of the least efficient bank, no bank runs, let alone contagion. When the realized return is just below the least efficient bank such that only a small fraction of the banking industry has liquidity problems, the liquidity demand can be absorbed by the securities market without causing liquidity crunch and hence no contagion occurs. However, when the realized return is so low that a significant amount of banks are facing liquidity problems, a liquidity crunch will occur and the market price of the risky asset will be driven down by the liquidity constraint. The drop of the market price will induce more banks on the run, thus starting the contagion process.

Given the realized return to the risky asset, bank runs and their contagion are determined by the distribution of the threshold values of the banks. Since according to (8), the distribution of the banking cost, or to be exact, the realization of the banking cost composition, has a direct mapping on the distribution of the threshold value, it also has a direct effect on contagion. The uniform distribution assumed in the numerical example is just one example. Suppose there are two clusters of banks with one cluster efficient (with low threshold values) and the other inefficient (with high threshold values). Then a moderate realized return (between the two threshold values) will cause runs for banks in the inefficient cluster. If the inefficient cluster is big enough to cause serious liquidity problems, or if the threshold values for the two clusters are close to each other, runs will be likely to spread to the efficient cluster. On the contrary, if the threshold values for the two clusters are far apart and the liquidity problems caused by the inefficient cluster are moderate, a moderate drop in the market price of the risky asset may not be enough to initiate runs in the efficient cluster. It is also possible that contagion happens just within the inefficient cluster and stops propagating to the efficient cluster due to the large gap between their threshold values. Contagion can happen at different degrees depending on the distribution of the banking cost structure.

Demographic composition of consumers affects the distribution of the threshold values by varying the investment decisions and the optimal contract. As illustrated in Figures 2 and 3, a large proportion of patient consumers imply a high investment rate in the risky asset and, given the distribution of the banking cost, a more clustered distribution of the threshold value. As a result, holding other things equal, contagion is more likely to happen when there are more patient consumers.

An alternative way to explain this phenomenon is that while the role of impatient consumers is fixed: they withdraw from the bank at period 1 no matter what, the behavior of the patient consumers is *a priori uncertain* and adds potential risk of bank run. It is the decision of the patient consumers that governs whether there is a run or not. When a large fraction of consumers in a bank are patient consumers, their decision of running a bank will have a larger effect on the relative balance of liquidity demand and supply and is thus more likely to be contagious. In addition, as a simultaneous endogenous decision, banks will invest more in the illiquid risky asset (X) as the fraction of patient consumers increases (Figure 2). This implies that the liquidity of the market (safe asset investment) will be smaller. As a result, a liquidity crunch is more likely to happen in case of an individual bank run.

This point can be clearly illustrated by an extreme example where all consumers are patient consumers ($\alpha = 0$) and they are close to risk-neutral[11] such that only a marginal amount of the money (say, \$1 out of \$1 billion) is invested in the safe asset. This is a market with extremely low liquidity. It is fine as long as no investor wants to withdraw at period 1; however, even when a very small fraction of investor wants to withdraw, the bank will not have enough safe asset to pay the promised consumption $\overline{c}$ and have to be liquidated. Yet, since there is just one dollar of the safe asset in the market, the risky asset can only be liquidated at a price next to nothing. At a market price close to zero, consumers of all banks will decide to run. The banking industry will crush with no other choices due to the extremely low liquidity.

In a similar vain of argument, a higher expected return on the risky asset at period 0 will also increase the investment in the risky asset X and thus decreases the liquidity of the market. Contagion is more likely to happen once the realized return is low enough to initiate a bank run. This effect can also be seen from the partial derivative

$$\frac{\partial R_i^*}{\partial C_i} = \frac{1}{X}.$$

The slope of R_i^* versus C_i is decreasing with increasing X. Given the distribution of banking cost, the threshold values R_i^* will become more clustered with more investment in the risky asset.

The last two points have an interesting implication: A good prospect for the overall the economy, as captured by a higher expected return to the risky asset, and

[11] Under the assumption of the previous example (with log utility), the allocation to the risky asset $X = 6.49$ out of \$10 when all consumers are patient ($\alpha = 0$). The optimal contract for early consumers $\overline{c} = 9.49$. We need to make the consumers closer to risk-neutrality to increase the risky investment to close 99% of the endowment.

a young society, where a large fraction of consumers are willing to wait for a longer period, are generally more susceptible to contagion once a crisis begins.

In another perspective, when a society is relatively old and pessimistic about the economic prospect, a relatively large fraction of the investment will be in the safe assets, the distribution of the threshold value becomes more disperse: the threshold value for inefficient banks becomes even higher and the threshold value for efficient banks becomes even lower. As a result, inefficient banks are difficult to survive but their crises are less likely to have a contagious effect.

Proposition 3 *Contagion can be either due to a very low realized return to the risky investment (recession) or due to a clustered distribution of bank threshold values for bank runs. The distribution of bank threshold values depend upon the distribution of the banking cost, the demographic composition, and the economic prospect. A clustered banking cost distribution, a high proportion of patient consumers, and an optimistic economic forecast all contribute to the clustering of bank threshold values, and thus to contagion.*

The proposition leads to two corollaries that illustrate the relationship between the business cycles of an economy and the proneness of contagion. In particular, the corollaries depict the potential "curse" of a sustained period high economic growth and the natural selection role played by a recession.

Corollary 1 *Contagion is more likely to happen after a period of high economic growth, when consumers become more optimistic and thus more willing to wait for a longer horizon and when banks become more optimistic and thus invest more in the risky assets.*

Corollary 2 *A period of recession acts as a natural selection mechanism when all banks and consumers become more conservative. Banks can be more easily distinguished from each other by their quality of banking: Less efficient banks can hardly survive while efficient banks can survive easily and are unlikely to be infected by contagion.*

After a sustained period of high economic growth, it is natural for both the banks and consumers to become more optimistic on the prospect of the economy. As a

result, more will be invested in the risky and illiquid sector of the economy. With a resultant tight liquidity supply, once the economy slows down and some bank falls, a contagious panics is most likely to follow.

On the other hand, after the economy is hard hit by a recession, investors become more conservative and their view of the economy more pessimistic, the result is a more conservative investment and more abundant liquidity supply in the market. As a result, the threshold values of different banks are more dispersed. Less efficient banks can hardly generate high enough return to overcome their very high threshold values while efficient banks are much easier to survive with much lower threshold values.

4 Application, Interpretation, and Extensions

The model above illustrates the key point that liquidity crunch, coupled with imperfect information, is the key culprit for a crisis to be contagious. However, the model itself is very stylized compared to the real world complicacy, and implications may very likely change with modifications on the set up. Cautions must therefore be applied when trying to interpret and/and apply the model to real time situations. Nevertheless, it is interesting to explore some of its implications and possible extensions.

4.1 The Asian crises

The 1997 Asian crises spread to a whole range of countries with sharply different characteristics. The exact reasons why each country was in trouble varied from country to country. Yet, our model may have captured, in a very stylized way, the two major common characteristics of these countries that made the crises so contagious. One is the long period of high economic growth in this area before the crises; the other is the extremely high saving rates. The former lead investors to form an optimistic forecast about their investments while the latter may imply that the majority of the population in that area are more willing to hold a long-term perspective, i.e. "patient" in terms of the model. Both, as proposed in Proposition 3, contribute to the extremely high investment rates observed in these countries, particularly in the illiquid sector such as real estate. As a result, once the economy began to slow down (low realized return R), the problems of some debt-driven banks

(high banking cost C_i) were exposed and needed to be closed. Yet the high investment rates in the illiquid sector implies that the aggregate liquidity of the market is small. Liquidity crunch was bound to happen once a crisis started. Contagion became inevitable.

Asian countries had experienced several decades of outstanding economic performance. annual GDP growth in the ASEAN-5 (Indonesia, Malaysia, the Philippines, Singapore, and Thailand) averaged close to 8 percent over the last decade. Indeed, during the 30 years preceding the crises per capita income levels had increased tenfold in Korea, fivefold in Thailand, and fourfold in Malaysia. Moreover, per capita income levels in Hong Kong and Singapore now exceed those in some industrial countries. In 1996, the year prior to the crises, most countries in the region experienced a slowdown in GDP growth. For example, the GDP growth rate in Korea fell from 8.9% in 1995 to 7.1% in 1996; Thailand fell from 8.7% to 6.7%; Malaysia fell from 9.5% to 8.2%; Singapore from 8.8% to 7.3%. While the average growth rate in 1996 was still high among these countries, compared to other regions of the world, the slowdown of these Asian economies does play a role in exposing the problems in the banking sector.

Unlike many Latin American countries with structurally low saving rates, the Asian countries were characterized by very high saving rates, in many cases above 30% of GDP (and in some cases above 40% of GDP) through the 1990s. These high saving rates provide sufficient cheap capitals for banks to make investments and to exploit profits. These high saving rates correspond to a small α in our model: most consumers are patient and therefore the liquidity constraint is loose in the optimization.

In addition, in the early 1990s, attracted by the great prospect in the Asian countries, a large amount of capitals flow into the Asian markets. Until the recent crisis, Asia attracted almost half of total capital inflows to developing countries, nearly $100 billion in 1995.

Faced with high saving rates and large capital inflows, the investment rates in these Asian countries are extremely high through the 1990s. In many countries, the investment rates are well above 30% of GDP and in some cases above 40% of GDP. The efficiency of the investment, however, has been deteriorating before the crisis. Another problem with the high investment rates is that an important fraction of the investment boom in these Asian countries was in the non-traded sectors such as commercial and residential construction and other non-traded services rather in

the traded sector. The early 1990s boom in residential and, especially commercial, construction lead to a glut of vacant buildings that led to the asset deflation of 1997.

As the economy cooled down, banking problems were exposed. Some debt-ridden banks in Thailand were forced to close. The once-abundant liquidity, mainly due to short-term foreign capital, suddenly disappeared. The flight of the short-term foreign capital, often termed as "hot money," aggravated the liquidity problem. Contagion became inevitable. As we have observed, the impact of the Asian crises was far beyond the banking sector and the range of the Asian countries. In late October, the whole world financial market was heavily disturbed by the waves of the Asian crises. The world stock market took a big tumble, as depicted in Figure 1.

While the interpretation is interesting, it surely misses many pieces of the real situation. For one, we assume in our model that all banks invest in the same project, which is a convenient normalization. Another assumption is that all banks participate in the same securities market, which is a key ingredient for contagion to happen because consumers observe prices from this market and liquidity of all banks are channeled through this market. Both assumptions strengthen the ties between the banks and make contagion from one bank to another more likely. While it may be a reasonable assumption for contagion from one bank to another within one country, the links between countries are usually not as strong and, therefore, international contagion may also not be as sweeping as indicated in the model. However, the model does imply that a common negative shock on the investments of different countries and close links between these countries through either securities market or trade, do increases the likelihood of contagion. Indeed, the empirical work of Rigobon (1999) does find that during the three episodes of 1994 Mexican crisis, the 1997 Asian crises, and the 1998 Russian crisis, contagion was more likely to happen between countries with strong GDP correlations and/or other trade ties. Eichengreen, Rose, and Wyplosz (1996), using data for 20 industrial countries from 1959 through 1993, also find that trade links have the greatest explanatory power on contagion effects. While our model abstracts from any international macroeconomic details, such as exchange rates, international trade, and foreign investment and so on, it does demonstrates in a stylized manner how the liquidity crunch can trigger the occurrence of contagion through a common shock to investment (low R) and an integrated securities market.

4.2 The role of imperfect information

Contagion in the model hinges critically on the assumption of imperfect information: consumers only observe the market price of the risky asset and base their decision on the market price and the banking cost. When the consumers observe the true return, there will still be individual crises due to low realized return, but there will not be any contagion effects. In fact, in time of liquidity crunch, consumers of some banks may even revert their running decision since they would know that by staying one more period they can actually benefit from the liquidity crunch by buying the risky project as a discount and therefore earn some extra returns (liquidity premium) on the project.

Suppose at period 1, the consumer knows the true return to the risky asset R as well as observing its market price. They would know that in case of liquidity crunch, their bank will sell all its residual liquid (safe) asset for the risky asset to earn a liquidity premium. The consumption of patient consumers at period 2 in case of liquidity crunch ($P < R$) hence becomes

$$c_2(P, R) = \frac{\frac{R}{P}(L - \alpha\overline{c} - C_i) + RX}{1 - \alpha}.$$

Compare this with period 1 consumption $\overline{c}$, we have the condition for the patient consumer to wait:

$$P \leq \frac{R(L - \alpha c - F)}{(1 - \alpha)c - RX}.$$

It says that patient consumer will wait until period 2 when either the realized return is high enough, or the liquidity crunch is severe enough for them to earn a big liquidity premium. The liquidity premium (LP), that is, the extra return earned by a patient consumer due to liquidity crunch, is

$$LP = \frac{(R - P)}{P}\frac{(L - \alpha\overline{c})}{1 - \alpha}.$$

4.3 Multiple risky assets

To focus on the effect of contagion across different banks (captured by different banking costs) in the banking industry, we have been assuming that there is just one risky asset. As a result, investment from different banks generates the same return. This setup can be easily extended to multiple risky assets, from which we can see the effect of contagion across different asset: the market prices of different

assets will become correlated under contagion even when the underlying technology is totally uncorrelated.

Now suppose there is a pool of risky assets that different banks can choose. Let R_i denote the return to the portfolio of risky assets chosen by bank i. R_i can sure be correlated across banks. In the above modeling of one risky asset, the correlation is 1. For the following analysis, we will instead assume zero correlation between R_i of different banks to illustrate the point that market prices of these risky assets (or portfolios), $P_i(R)$, can become correlated even if the underlying technologies are totally uncorrelated. To keep the bank's decision at period 0 identical, we assume that R_i has the same distribution for all i, although their realization can be different at period 1.

Similarly, we now assume that consumers in period 1 observe the market price of each asset but not their underlying return of the technology. We still normalize the market prices at time 0 for all assets to 1. At times when there is no bank run or liquidity crunch, market prices and asset returns coincide with each other at period 1. In case of liquidity crunch, the market price falls below the return to the technology.

The bank's decision in (6) will be modified only slightly, replacing the market price $P(R)$ with $P_i(R)$:

$$\begin{array}{lll} \max_{\{\overline{c},(L,X)\}} & & E\left[\alpha g(c_1(R)+(1-\alpha)g(c_2(R))\right] \\ \text{s.t.} & \text{(i)} & L+X\geq E; \\ & \text{(ii)} & c_1(R)=\left\{\begin{array}{ll} \overline{c} & \text{if} \quad P_i(R)\geq R_i^* \\ L+P_i(R)X-C_i & \text{if} \quad P_i(R)<R_i^* \end{array}\right. ; \\ & \text{(iii)} & c_2(R)=\left\{\begin{array}{ll} (L-\alpha\overline{c}+RX-C_i)/(1-\alpha) & \text{if} \quad P_i(R)\geq R_i^* \\ L+P_i(R)X-C_i & \text{if} \quad P_i(R)<R_i^* \end{array}\right. . \end{array} \tag{12}$$

The securities market clears based on the following condition:

$$\begin{array}{lll} \text{(i)} & P_i(R)=R_i \text{ for all } i & \text{if} \quad R_i<R^0 \text{ or } R_i\geq R_i^* \text{ for all } i; \\ \text{(ii)} & \left\{\begin{array}{lll} L_s & = & X\sum_{i=1}^k P_i(R) \\ \frac{P_i(R)}{R_i} & = & \frac{P_j(R)}{R_j} \text{ for all } i,j. \end{array}\right\} & \text{if} \quad R^0<R_i<R_i^* \text{ for } i=1,\cdots,k. \end{array} \tag{13}$$

It says that when there is enough liquidity in the market, the market price equals the real return to the technology. In case of k bank runs and liquidity crunch, the market price is restricted by the liquidity supply of the healthy banks L_s. Since to the healthy banks, the liquidated risky assets are bargain opportunities, they will

try to offer each other their risky investment for the liquidated risky assets to the point that the ratio of market price to return is the same for all risky assets. As a result, the market prices of the risky assets in the healthy banks will also fall proportionally.

The previous example can go through will little modification if we assume that all bank's risky investments, though independent from each other, have the same realization: $R_i = R$. Using the same number from the previous example, we have the threshold values for these banks:

$$R_i^* = [1.14; 1.08; 1.02; 0.96; 0.90; 0.84].$$

Similarly, the no-bank run healthy period happens when the realized return $R_i > 1.14$; the individual bank run with no contagion happens when the realized return falls between $[1.08, 1.14]$. When the realized return is below 1.08, the first two banks go bankruptcy and generate liquidity crunch: $R^0 = 0.94 < R_i = 1.08$. Since all risky investments happen to have the same realized return, the market price will also fall to the same level: $P_i(R) = 0.94$ for all i, which is then below the threshold value of the third and fourth banks. Liquidity crunch aggravates, market prices of assets falls further and the whole banking industry tumbles.

Note that although the realization of the returns to the risky investments of different banks are assumed to be independent, they move (to be exact, they fall) in the same proportion when there is liquidity crunch and thus generate positive correlation between each other.

When the realized returns are different across banks, which should be the case in general, the analysis is a bit complicated. Contagion becomes more likely when, say, more efficient banks (with lower banking cost) have low realized returns while less efficient banks have higher realizations. On the other hand, bank runs are more likely to stop at the worst banks when the less efficient banks happen to have low realizations and more efficient banks happen to have high realizations. The independent investment opportunities add more randomness to whether banks' differences are bigger or smaller and thus contagion is more or less likely. Additional correlation between asset returns and between banking cost and returns will alter the specific scenario but will not vary the general classification of the three phases: (1) healthy phase where there is no bank run, (2) individual bank run phase where there is no contagion, and (3) banking panics where contagion occurs. Of course, depending on the distribution of the cost structures across banks and the distribution of realized returns, partial contagion is also possible.

Here although we allow banks to invest in different risky projects, we still assume that they are traded in an integrated securities market. While this is more likely the case in a domestic market, internationally the markets are more or less separated. Those listed in Hong Kong stock exchange are mostly different stocks from those listed in the New York Stock Exchange. Under our set-up, suppose there are two independent securities markets, each with a different group of banks with independent risky projects, then contagion would not be possible from banks in one securities market to banks in another. The reason is simple: since there is no trade between the two markets, prices in one market will not adjust to liquidation needs in another. Hence if we assume different countries have totally independent markets, contagion would be mostly domestic and hard to spread to foreign countries. However, we do observe contagion across borders like the case of the 1997 Asian crises, among many other crises. In reality, countries are connected one way or the other. International trade link is one, international portfolio diversification is another.

One way to link banks of different countries together, assuming independent securities markets, is to assume that the investments of banks from different countries are linked. Once scenario is that banks in country A are "levered" by banks in country B. That is, banks in country B invest some of their money in banks in country A. When a contagious crisis happens in country A, the investment returns for banks in country B also drop down and thereby lead a contagious run in country B. Another scenario is that monetary policies and business cycle in country A have an impact in country B's economy through international trade links. An example would be that a currency devaluation in country A may have a negative impact on country B's exports and therefore their real output. Therefore, a currency crisis in country A may put pressures on country B's currency and economic performance. In real life situations, there must be many factors, including economic, political, or maybe even psychological ones, that may link two countries together and lead to contagion from one country to another, and indeed, different contagious panics must have different causes. Our stylized model has no specific says on which is the exact mechanism for a specific crisis to become contagious. It just tells us that, with interlinked securities market or investment technologies, liquidity crunch, together with imperfect information, can generate contagious runs.

Remark 1 *A necessary condition for contagion to happen from one country to another is that these countries need to have either interrelated securities markets, or interrelated investments, or both.*

4.4 Contagion in the securities market: Market crashes

As the above analysis illustrates, contagion in the banking industry also spreads to the securities market. At normal times when there is no bank run and there is enough liquidity, the market price of each investment reflects its real return. The correlations between the production technologies are reflected by the correlations in their market prices. However, when there is a contagious bank run, the prices of all the risky assets involved in the bankrupt banks will fall due to the liquidity crunch, as illustrated in condition (13). As a result, the correlations between asset prices are higher in financial distress than in normal times.

Casual observation aside, measuring the correlation change between normal and crisis times has been proved not to be an easy manner. Das and Uppal (1997) find that the correlation of the different stock indices increases with the volatilities of these indices. It has, however, been shown by many authors that there may an upward bias in these measures of correlations: Conditional on increased volatility, the correlation between asset prices increases even when the unconditional correlation does not change. Forbes and Rigobon (1998) take that explicitly into account and find that the increases in correlation become much smaller. However, the adjustment is made while assuming independence of shocks from the state variables. As Foresi pointed out in a discussion, the direction of the bias is actually ambiguous and can be downward when the shocks are correlated with the state variables. Rigobon (1999) went one step further to take care of problems of endogenous variables, omitted variables, and heteroskedasticity and construct a "structural change" measure to test whether there are structural change of either kind before and after a crisis. Issues remain on how to identify a crisis ex post and on whether it is still a valid measure when the shocks and common factors are correlated. Backus and Wu (1998) find that correlations between these higher moments tend to be higher than correlations between the second moments, which tend to capture the normal time market variations.

Remark 2 *Asset returns tend to have higher correlations during times of liquidity crunch.*

4.5 Ex ante expectation of having a bank run

In the numerical example we present, we assume, for ease of solving the problem, that ex ante banks do not expect to have bank runs of any kind. As a result, the

problem is solved by assuming that consumption for early consumers is always $\overline{c}$ and for later consumers is whatever is left. Also, since there is assumed to be no bank run *a priori*, the market price of the risky asset equals to its real return R.

If ex ante banks know that there is a probability of having bank runs and that the probability is related to his banking decision, we would expect the bank to alter his decision. In what follows, we will investigate how the bank's decision will be altered by the bank's ex ante expectation of a bank run and its contagion.

We keep assumptions on the utility function and the distribution of return the same as in the previous numerical example. But we simply further by assuming that banks make decision assuming an average cost: $C_i = \overline{C}$. Of course, the ex ante expectations regarding bank runs are also different.

First, we assume that ex ante a typical bank only expects two scenarios: (1) a contagious bank run, with probability p, such that the market price of the risky asset falls to zero and all consumers split $(L - C_i)$ and (2) no bank run, with probability $(1-p)$, when the market price equals the true return, early consumer consume $\overline{c}$ and late consumers consume whatever is left. We assume that the probability of having a contagious bank run, p, is exogenously given and is independent of the bank's decision. Banks' decision problem becomes

$$\begin{aligned}(I) \quad \max_{\overline{c},X} \quad & (1-p)E\left[\alpha \ln(\overline{c}) + (1-\alpha)\ln\left(\frac{(R-1)X - \alpha\overline{c} + E - \overline{C}}{1-\alpha}\right)\right] \\ & +p\ln\left(L - \overline{C}\right)\end{aligned}$$

where p is the probability of the contagious bank run and the expectation operator $\mathbb{E}[\cdot]$ is taken over the distribution of R. The two first order conditions are

$$\begin{aligned} E\left[\frac{1}{\overline{c}} - \frac{(1-\alpha)}{(R-1)X - \alpha\overline{c} + E - \overline{C}}\right] &= 0; \\ E\left[\frac{R-1}{(R-1)X - \alpha\overline{c} + E - \overline{C}}\right] &= 0; \end{aligned}$$

which are exactly the same as in the case where banks do not expect any banks. Bank decision will not be affected by bank runs that they have no control on.

With the simplification on the banking cost, the optimal deposit contract can be solved easily as: $\overline{c} = E - \overline{C}$. The investment decision X can be solved from the following equation:

$$\frac{X(\overline{R} - \underline{R})}{(E - \overline{C})(1-\alpha)} = \ln \frac{X(\overline{R} - 1) + (1-\alpha)(E - \overline{C})}{X(\underline{R} - 1) + (1-\alpha)(E - \overline{C})}.$$

However, in reality the probability of having bank runs depends crucially on the threshold value of a bank, which in turn is a function of banks' decision. Therefore, in reality, banks' decision and the probability bank runs are closely intertwined.

Now suppose there are still two scenarios: either there is a contagious bank run or there is no bank run at all. The probability of having a bank run, however, is no longer exogenously given but is determined by its threshold value R^*:

$$R^* = \frac{\overline{c} - L + \overline{C}}{X} = 1 + \frac{\overline{c} - E + \overline{C}}{X},$$

which is a function, in itself, of the bank's decision. The banks' decision problem is now

$$\begin{aligned} (II) \quad \max_{\overline{c},X} \quad & \int_{R^*}^{\overline{R}} \left[\alpha \ln(\overline{c}) + (1-\alpha) \ln \left(\frac{(R-1)X - \alpha\overline{c} + E - \overline{C}}{1-\alpha} \right) \right] dR \\ & + \int_{\underline{R}}^{R^*} \ln \left(L - \overline{C} \right) dR. \end{aligned}$$

This would be the case in the absence of the securities market. Without the securities market, the running bank cannot exchange its illiquid asset holdings for cash (liquid asset). Since its liquidation value is zero, it is worth nothing being liquidated at period 1. The two first order conditions are:

$$\begin{aligned} \int_{R^*}^{\overline{R}} \left[\frac{\alpha}{\overline{c}} - \frac{\alpha(1-\alpha)}{(R-1)X - \alpha\overline{c} + E - \overline{C}} \right] dR + \frac{1}{X} \ln \frac{(E - X - \overline{C})}{\overline{c}} &= 0; \\ \int_{R^*}^{\overline{R}} \frac{(1-\alpha)(R-1)}{(R-1)X - \alpha\overline{c} + E - \overline{C}} dR - \frac{\overline{c} - E + \overline{C}}{X^2} \ln \frac{(E - X - \overline{C})}{\overline{c}} &= 0; \end{aligned}$$

Since the probability of having a contagious bank run is now directly related to the bank's decision. It can be shown that the bank, expecting such a probability, tend to act more conservatively by offering a smaller consumption bundle $\overline{c}$ for early consumers.

Similar results hold when market liquidity is being taken consideration explicitly. Refer back to the securities market condition (7),

$$\begin{cases} \text{(i)} \quad P(R) = R & \text{if} \quad R < R^0 \text{ or } R \geq R_1^*; \\ \text{(ii)} \quad P(R) = L_s/(kX) & \text{if} \quad R^0 < R < R_k^*. \end{cases}$$

When there is liquidity crunch ($R^0 < R, R_k^*$) in the securities market, the market price of the risky asset is inversely proportionally to the risky investment X. Taking

this security market condition into consideration, we can assume that there are three phases for an average bank: (1) no bank run ($R > R^*$), (2) individual ban run ($R^* < R < R_2^*$) with asset price being inversely proportional to the risky investment: $P(R) = B/X$ (where B is some constant related to the aggregate market condition), and (3) contagious bank panics with the market price of the risky asset falling to zero ($R < R_2^*$). Further, we assume that the threshold value R^* is a function of the bank's decision while the contagion point R_2^* is exogenous: the effect of bank's decision on R_2^* is negligible. In such a setup, we can capture the role of liquidity by the assumption that $P(R)$ is inversely proportionally to X. Nevertheless, the problem is simplified to a great extent in regard to the effect of a bank's decision on contagion in general. An average bank' decision problem is now

$$(III) \quad \max_{\overline{c},X} \int_{R^*}^{\overline{R}} \left[\alpha \ln(\overline{c}) + (1-\alpha) \ln \left(\frac{(R-1)X - \alpha\overline{c} + E - \overline{C}}{1-\alpha} \right) \right] dR$$
$$+ \int_{R_2^*}^{R^*} \ln\left(E - X + B - \overline{C}\right) dR + \int_{\underline{R}}^{R_2^*} \ln\left(L - \overline{C}\right) dR$$

The two first order conditions are

$$\int_{R^*}^{\overline{R}} \left[\frac{\alpha}{\overline{c}} - \frac{\alpha(1-\alpha)}{(R-1)X - \alpha\overline{c} + E - \overline{C}} \right] dR + \frac{1}{X} \ln \frac{E - X + B - \overline{C}}{\overline{c}} = 0;$$
$$\int_{R^*}^{\overline{R}} \frac{(1-\alpha)(R-1)}{(R-1)X - \alpha\overline{c} + E - \overline{C}} dR - \int_{R_2^*}^{R^*} \frac{1}{E - X + B - \overline{C}} dR$$
$$- \frac{\overline{c} - E + \overline{C}}{X^2} \ln \frac{E - X + B - \overline{C}}{\overline{c}} = 0.$$

It can be shown that both early consumption $\overline{c}$ and risky asset holding X will be reduced, but not as much as in the precious example.

Proposition 4 *Anticipating that a bank run can happen and that the probability of having such a bank run is directly related to the threshold value of the bank, an average bank tends to make more conservative decisions by (1) offering a smaller consumption bundle $\overline{c}$ to early consumers and (2) investing less in the illiquid risky asset.*

The proof is given in the appendix for both cases.

5 Social Welfare Analysis and Policy Implications

Modeling contagion inevitably leads to discussions on policy issues. However, any policy discussion based on the model is speculative considering the stylized nature of the model. International Monetary Fund (IMF) has played a very active role in the recent financial crises in the 1990's and also has caused hot debate on some of the bail-outs as well as its operating procedures. See the report by Ito et al (1999) for a excellent survey of the common criticisms and reform proposals. Under the structure of our model, we will speculate the roles that can be played by an agency like IMF.

Before we can analyze the role of IMF, we need to clarify what IMF is, what kinds of power it has, and what it does. Usually, hot debates on where IMF has been doing a good job depends crucially on the assumptions made about IMF.

Here we define a *stylized* IMF who collects money as tax from banks at period 0 and then provide liquidity if needed at period 1. We also assume that IMF acts like a social planner who maximizes social welfare. We call it IMF while it may nothing to do with the real-life International Monetary Fund.

At period 1, if the economy is healthy and no bank needs to be liquidated, IMF needs to do nothing and will return the tax back to the banks who will pay back the consumers as specified by the deposit contracts. Suppose some banks do need to be liquidated, what IMF can do is to provide them with extra cash in return for the risky project. As long as IMF has enough cash, these banks can exchange their risky project for RX of cash. However, the amount of cash available to IMF is finite as the endowments of consumers are finite and IMF's reserve is just a fraction of them. Hence, when IMF does not have enough cash, IMF can only provide these running banks with less cash than RX. Say, IMF will provide cash equal to PX with $P < R$ when IMF does not have enough cash.

In what follows, we will investigate what such an IMF can do to increase social welfare. But before we do that, we first define a benchmark case of optimality.

5.1 An optimal benchmark

As a benchmark, we assume that at period 0 banks can offer state-contingent contracts which specifies that when $R > R_i^*$, early consumers receive $c_1(R) = \overline{c}$ and

late consumers receive

$$c_2(R) = \frac{L - \alpha\overline{c} - C_i + RX}{1-\alpha}.$$

When $R \leq R_i^*$, both early and later consumers receive the same package:

$$c_1(R) = c_2(R) = L - C_i + RX.$$

Furthermore, R_i^* is defined as

$$R_i^* = \frac{\overline{c} - L + C_i}{X}$$

such that $c_2(R) \geq c_1(R)$ always holds. For this to become a viable contract, we must also assume that consumers can verify the realized state, which includes the realized return R and the banking cost C_i.

Under such a set-up, patient consumers do not have an incentive to withdraw early: they either strictly prefer to consume at period 2 or are indifferent between consuming at period 1 or 2. Hence the bank does not to liquidate as long as it has enough cash at period 1 to cover impatient consumers, that is,

$$L > \alpha\left(L - C_i + RX\right)$$

or

$$L > \frac{RX - C_i}{1-\alpha}$$

which serves as a liquidity constraint in the investment decision.

When $R \leq R_i^*$, we can think of the bank as having a crisis but the crisis is socially optimal in the sense that it does incur any deadweight cost.

Under such a set-up, there will be no liquidation need and therefore no need for the existence of the the securities market.

Ex ante, banks offer a deposit contract $\overline{c}$ and make investment decisions that maximize the aggregate utility of consumers. This optimal benchmark can be summarized as follows:

$$\begin{array}{lll}
\max_{\{\overline{c},(L,X)\}} & & \mathbb{E}\left[\alpha g(c_1(R) + (1-\alpha)g(c_2(R))\right] \\
\text{s.t.} & \text{(i)} & L + X \geq E; \\
& \text{(ii)} & c_1(R) = \begin{cases} \overline{c} & \text{if} \quad R > R_i^* \\ L - C_i + RX & \text{if} \quad R \leq R_i^* \end{cases}; \\
& \text{(iii)} & c_2(R) = \begin{cases} (L - \alpha\overline{c} + RX - C_i)/(1-\alpha) & \text{if} \quad R > R_i^* \\ L + RX - C_i & \text{if} \quad R \leq R_i^* \end{cases}; \\
& \text{(iv)} & R_i^* = \frac{\overline{c} - L + C_i}{X}.
\end{array} \tag{14}$$

5.2 The sub-optimality of contagion

Return to our original set-up where banks can only offer a standard deposit contract $\overline{c}$ and consumers do not know the true return at period 1 but just observe the market price. Assume that ex ante banks do not expect contagious bank runs, then $P = R$ and their contract offer of $\overline{c}$ and investment decision is ex ante optimal because it coincides with the social optimal decision in (14).

Ex post, as long as there is no liquidity crunch, the securities market will channel the money between the liquidating banks and the healthy ones with the market price fully reflect the true return: $P(R) = R$. The consumption bundles are the same as in the optimal case. However, when liquidity crunch occurs, the risky project of the running banks need to be liquidated at a discount. Consumers in these banks will therefore receive a consumption bundle smaller than the optimal one. In case when the contagion is partial and some banks can survive the liquidity crunch, it amounts to a wealth transfer from running banks to healthy banks. Even when consumers diversify ex ante among all the banks, there remains a wealth transfer from impatient consumers to patient consumers because the loss of the impatient consumers in the running banks will not be able to be compensated in the healthy banks, as they will receive $\overline{c}$ as promised. This wealth transfer amounts to a loss in social welfare unless consumers have linear utilities (risk-neutral).

In case of complete contagion, nobody survives to enjoy the fruit of the risky project. The social welfare loss is obvious. The risky project has to be abandoned due to shortage of liquidity.

On the other hand, if we assume that ex ante banks expect contagious bank runs and therefore invest more in the safe asset and less in the risky asset as is shown in Proposition 4, then the investment decision is not socially optimal. Intuitively, the potential existence of contagion makes liquidity a more important factor. It therefore amounts to an optimization problem under a tighter liquidity constraint and therefore is suboptimal.

5.3 The role of IMF

The IMF defined above essentially plays the role of the securities market. It channels liquidity from healthy banks to those running banks. However, since the liquidity reserve of IMF is finite, when there is a significant number of banks on the run, it

will not be able to provide enough cash to liquidate the risky projects at its true long-term value. As a result, IMF can only provide them cash at a discount $P < R$.

Whether contagion will occur under such a scenario will depends crucially on one's assumption. Suppose consumers as before do not know the true return but observe such a discount factor P, the situation becomes exactly the same as with the securities market replacing IMF. There will not be any social welfare gain by introducing IMF because everything IMF does ex post is performed by the securities market. Ex ante, through taxation, IMF presumably can influence banks' decision on deposit contract and investment. However, since banks are already maximizing the aggregate utility of consumers, IMF's tax scheme will not be able to improve the investment efficiency unless IMF has a different (and more precise, say) information set than the banks.

An example of this information difference is that while banks make competitive decisions without considering contagious runs in an aggregate level, IMF may have a bigger picture about the aggregate liquidity of the market and the possibility of having bank runs. As a result, from the perspective of IMF, it is optimal for the banks to invest more conservatively than they want to and IMF can force a more conservative investment by ex ante taxation. For example, suppose banks's investment decision on liquidity is L_1 while IMF's decision is $L_2 > L_1$. Then at period 0, IMF can tax L_2 from all banks and return the same amount back to them at period 1. Banks will invest all their residual endowments to the illiquid risky project and thus falls exactly into the optimal decision of the IMF. With different information set, however, the deposit contract $\overline{c}$ may also be different and taxation itself may not able to make their decisions converge.

In some other papers like in Allen and Gale (1998), a social planner, or a "lender of last resort" is assumed to have some exogenously given "extra liquidity." The lender of last resort is assumed to always have enough liquidity to liquidate the risky project as its full value. That obviously will solve the "liquidity crunch" problem and therefore kill contagion. The question is where this extra liquidity come from. To completely kill liquidity crunch at any scenarios, this lender of last resort has to have enough cash to cover all banks' risky project. Furthermore, this money cannot come from taxation from these banks because the indifference of patient consumers between consuming at period 1 or 2 implies that all the endowments have to be invested in the safe asset to kill liquidity crunch at all times. This amounts to the *narrow banking proposal*," the idea that banks should be required to back demands deposits entirely by safe short-term assets. Wallace (1996) using a similar Diamond and Dybvig framework to show that such an proposal is neither innocuous nor

desirable and that using narrowing banking to cope with banking system illiquidity eliminates the role of the banking system.

In reality, however, the economy is not as efficient as in our model: The securities market may not exist or may not be as complete and integrated; banks may know more about the project than consumers at period 1 but they may not know everything as assumed in the model. Then IMF may be able to play a positive role to increase the efficiency on both ends either by direct participation of bail-outs or by regulation. In general, we argue that IMF can play three functional roles: (i) providing information, (ii) playing the function of a securities market (channeling liquidity), and (iii) coordinating.

5.3.1 Information

As discussed in the previous section, information plays a crucial role in contagion. If consumers observe the true return at period 1, there may still be individual bank runs but no contagion will occur. And ex ante banks can offer deposit contract that depends on the realized return and banking cost and therefore kills runs and contagion altogether. This is the optimal benchmark defined in (14).

A social planner can help in the direction of information in many different ways. In the case of IMF, in many of its recent bail-outs of the emerging markets countries, IMF often attach a string of conditions on the bail-outs. Among them include conditions that require these countries and banks in these countries to do western-style financial structural reforms. One example is to require them to do appropriate disclosure of their operations to increase the transparency of these countries. As illustrated in our model, information is a crucial element in contagious panics. The more information available to the consumers, the more they know about the true performance of these firms, the less likely they will decide to run. Casual observation also indicates that contagious runs are more likely to happen in emerging markets where the markets are less transparent and less likely in the industralized countries such as the United States where sophisticated rules on disclosure have made firms more transparent to the society. Therefore, reforms in the direction of transparency shall reduce the likelihood of contagious panics and increase social welfare, at least in the long run.

In the model, we assume that consumers only observe the market price while banks know the true return. However, in reality, imperfect information is all relative.

Banks, with the resources they have, may know more about other banks' investment performance they consumers do, but they may not know everything. Presumably, different banks may also have different information set. Imperfect information on the bank's side will hinder the trade in the securities market and will therefore make the liquidity transfer between banks less smooth. As mentioned in the introduction, during the 1907 crisis, J.P. Morgan, as a private investment banker, not only raised the money necessary for the bail outs, but also sent his associates to do the auditing job on those near bankrupting firms, which is the way to find out the true performance of these firms. There were many other banks which might not have the power to raise the money, nor the resource to do the auditing work.

Under such a scenario, a centralized planner like IMF might be able to collect more resources together to find out more about the running firms or countries and decide which one to be bailed out and which one to be left fall. As a result, IMF can perform functions that any single bank may not be able to perform.

5.3.2 Securities market

In the model, we assume the existence of an integrated securities market where all the banks can rebalance their portfolio. This might be reasonable assumption for a domestic market but the international securities market may not be as integrated in our model. Under such a scenario, the IMF can play the function of the securities market: when one country has liquidity problems, IMF can borrow money from other healthy countries and lend the money to the country in trouble and bail it out. This is, to some extent, the source of the "extra liquidity" of IMF in the mind of many authors.

5.3.3 Coordination

Sometimes, even with the existence of a securities market, some kind of coordination needs to be done before a bail out can actually occur. For example, suppose a country is in trouble and needs 5 billion dollars to be bailed out and any amount less than this figure will not be enough to keep it float. Suppose an investment bank has 1 billion dollars and he knows that it is a good investment if the country can be saved from collapsing. However, the investment bank will not invest its 1 billion dollars in the country in trouble unless it knows that 4 billion dollars are going to be chipped in by other investment banks. Sometimes, a securities market may

have a coordination failure in the sense that nobody is willing to take the trade for fear that others may not come in although it would have been a good investment had everybody acted together. An social planner like IMF will be able to solve the coordination problem by collecting all the money and decide whether it is enough to bail a firm/country or not and then take an action that is optimal for the society as a whole.

In the 1907 financial crisis, J.P. Morgan also played a similar coordinating role: he did not bail out those firms using his own money, but instead used his power to gather money from all major banks in New York. These banks either trusted his judgement or could not refuse him for whatever reasons. With the money, he was left to the coordinating work and information processing. Had these banks acted separately, they might not be able to act in harmony to bail these firms out. Also, referring back to the information function, it may be too time consuming for each bank involved to do a separately auditing job for the information. Centralization through J.P. Morgan made the bail out more efficient and ultimately feasible. IMF may have also been playing a similar rule.

5.4 Speculations on macroeconomic monetary policy

Since the model has no macroeconomic or monetary specifications, any discussion to this end would be even more speculative. Nevertheless, it may provide the following tentative implications one needs to pay attention to in policy making:[12]

- Since *liquidity crunch* is the key culprit of the contagious runs, the government may be better off having a loose monetary policy when detecting a likely contagious run in the country. The model, however, has little says about the exact exchange rate policy, which is an interesting area of research by itself.
- In time of liquidity crunch, closing out insolvent banks and forcing immediate recapitalization may aggravate the liquidity crunch and may therefore worsen the problem. The structure similar to that of the Chapter 11 rules in the United States might help — freezing debt-payment for a "restructuring period" may give some "inherently good" firms a period of breath for them to recover from pure temporary liquidity squeezes.

[12]This part benefits greatly from the discussion of Richard Portes at the 1999 NBER Summer Institute.

- Due to the important rule of information in the propagation of crises, it is always important to push for appropriate financial disclosure and transparency. A special attention may be the appropriate disclosure and measure derivative trading and other related "off-balance sheet activities", which incur great risk to the involved firms but may not show up in the traditional financial reports.

6 Final Thoughts

The model in this paper takes the banking industry as an example and illustrates how contagion can happen in the banking industry and go beyond the banking sector. Similar contagious events can happen in other parts of the financial markets now that the banking product, prototyped in the model by the standard deposit contract, is not unique any more.

The key feature that results in a bank run and its contagion is the complementarity nature of banking depositors' behavior, which drains the liquidity of the market. As price falls, patient consumers run and banks sell the risky asset, which aggravates the liquidity problem. The reason for this behavior is the liquidity constraint inherent in the banking practice. Whether a financial market has the potential risk of having a contagious "market run" depends on whether the structure of that market can generate this type of "running behavior" that is strong enough to drain the liquidity of that market and related markets. Commonly cited features that share similar characteristics of strategic complementarity include feedback trading, dynamic hedging, and portfolio insurance.

Once a crisis begins, how far its contagion can go depends on the aggregate liquidity of the market. Illiquid markets or markets where liquidity can come and go easily are more susceptible for contagious attacks. For example, high investment in the illiquid sector reduces the aggregate liquidity of the market. Another example is the heavy borrowing on short-term foreign capitals, which usually flow in and out easily. In particular, short-term foreign capitals tend to flow in when liquidity is abundant and flow out when the market needs them most. As a result, its inflow in good times creates a "fake" aroma of high liquidity and induces overinvestment in the illiquid sector while its outflow in bad times aggravates the liquidity crunch and the contagious effect of a crisis.

Another key ingredient for a crisis to be contagious is imperfect information.

The model also has important implications for both policy makers and securities designs. A sound policy should be directed to increase the market liquidity without sacrificing investment efficiency; a well-designed security, on the other hand, should increase the liquidity of the market without generating new sources of complementarity behaviors which can potentially drain the liquidity of the market.

A Proofs of Propositions

A.1 Proof of Proposition 4

Case I: When an average bank does not, ex ante, expects runs, or the run is independent of the bank's decision, the optimal deposit contract is given by

$$\overline{c} = E - \overline{C}.$$

The optimal investment decision X is solved from the following equation

$$\ln \frac{X(\overline{R}-1)+(1-\alpha)(E-\overline{C})}{X(\underline{R}-1)+(1-\alpha)(E-\overline{C})} = \frac{X(\overline{R}-\underline{R})}{(E-\overline{C})(1-\alpha)}. \tag{15}$$

There are multiple solutions to this equation. Obviously but trivially, $X = 0$ is a solution. We will focus on the solution that falls in the range

$$X \in \left(0, \frac{(1-\alpha)(E-\overline{C})}{(1-\underline{R})}\right).$$

Within this range, the left hand side of the equation is convex and upward sloping and goes from 0 to infinity. The right hand side of the equation is linear in X.

Case II: When an average bank expects a contagious bank run that depends on its threshold value, we will show, by comparing to the above solution, that the bank becomes more conservative by (1) offering less to consume for early consumers and (2) investing less in the risky asset.

Integration over the two first order conditions yield:

$$\begin{aligned} \frac{\alpha}{\overline{c}}(\overline{R}-R^*) - \frac{\alpha(1-\alpha)}{X}\ln\frac{X(\overline{R}-1)-\alpha\overline{c}+E-\overline{C}}{X(R^*-1)-\alpha\overline{c}+E-\overline{C}} & \\ +\frac{1}{X}\ln\frac{E-X-\overline{C}}{\overline{c}} &= 0; \quad (16) \\ \frac{1-\alpha}{X^2}\left[X(\overline{R}-R^*)+(\alpha\overline{c}-E+\overline{C})\ln\frac{X(\overline{R}-1)-\alpha\overline{c}+E-\overline{C}}{X(R^*-1)-\alpha\overline{c}+E-\overline{C}}\right] & \\ -\frac{\overline{c}-E+\overline{C}}{X^2}\ln\frac{E-X-\overline{C}}{\overline{c}} &= 0. \quad (17) \end{aligned}$$

From (16), we have

$$\ln\frac{X(\overline{R}-1)-\alpha\overline{c}+E-\overline{C}}{X(R^*-1)-\alpha\overline{c}+E-\overline{C}} = \frac{X(\overline{R}-R^*)}{\overline{c}(1-\alpha)} + \frac{1}{\alpha(1-\alpha)}\ln\frac{E-X-\overline{C}}{\overline{c}}. \tag{18}$$

Substitute (18) into (17) and rearrange, we have

$$\overline{c} - E + \overline{C} = \frac{(E - \overline{C})\overline{c}(1-\alpha)}{X\alpha(\overline{R} - R^*)} \ln \frac{E - X - \overline{C}}{\overline{c}}. \tag{19}$$

Since $E - X - \overline{C} < \overline{c}$, we have $\overline{c} - E + \overline{C} < 0$, or

$$\overline{c} < E - \overline{C},$$

which says that the promised consumption for early consumers is smaller than in the case where no bank run in expected.

Assuming $\overline{c} \approx E - \overline{C}$ and rearrange (18), we have

$$\ln \frac{X(\overline{R} - 1) + (1-\alpha)(E - \overline{C})}{X(\underline{R} - 1) + (1-\alpha)(E - \overline{C})} = \frac{X(\overline{R} - \underline{R})}{E(1-\alpha)} - A$$

with

$$A = \frac{X(R^* - \underline{R})}{E(1-\alpha)} - \frac{1}{\alpha(1-\alpha)} \ln \frac{E - X - \overline{C}}{E} - \ln \frac{X(R^* - 1) + (1-\alpha)(E - \overline{C})}{X(\underline{R} - 1) + (1-\alpha)(E - \overline{C})} > 0.$$

The left hand side is the same as in (15) while the right hand side is moved down by A. As a result, the solution X is going to be smaller.

Case III: When the market price of the risky asset, in case of liquidity crunch, is inversely proportionally to the risky asset holding, similar results can be obtained. Integration over the first order conditions and rearrange, we obtain the following two equations:

$$\begin{aligned} \overline{c} - E + \overline{C} &= \frac{(E - \overline{C})\overline{c}(1-\alpha)}{X\alpha(\overline{R} - R^*)} \ln \frac{E - X + B - \overline{C}}{\overline{c}} \\ &\quad + \frac{\overline{c}X(R^* - R_2^*)}{(\overline{R} - R^*)(E - X + B - \overline{C})}; \end{aligned} \tag{20}$$

$$\begin{aligned} \ln \frac{(\overline{R} - 1)X + (1-\alpha)(E - \overline{C})}{(R^* - 1)X + (1-\alpha)(E - \overline{C})} &= \frac{X(\overline{R} - R^*)}{\overline{c}(1-\alpha)} \\ &\quad + \frac{1}{\alpha(1-\alpha)} \ln \frac{E - X + B - \overline{C}}{\overline{c}} \end{aligned} \tag{21}$$

Compare (20) with (19), we see that (20) (1) has an extra positive (but small) term and (2) $E - X + B - \overline{C} > E - X - \overline{C}$. As a result, $\overline{c} - E + \overline{C} < 0$ although not as negative as in the previous case. Promised early consumption is smaller, though not as small in the previous case.

Compare (21) with (18), we see that the right hand side in (21) is a little bigger than in (18). So X is larger in this case than in the previous case, but still smaller than in (15), that is, the case when no bank run is expected.

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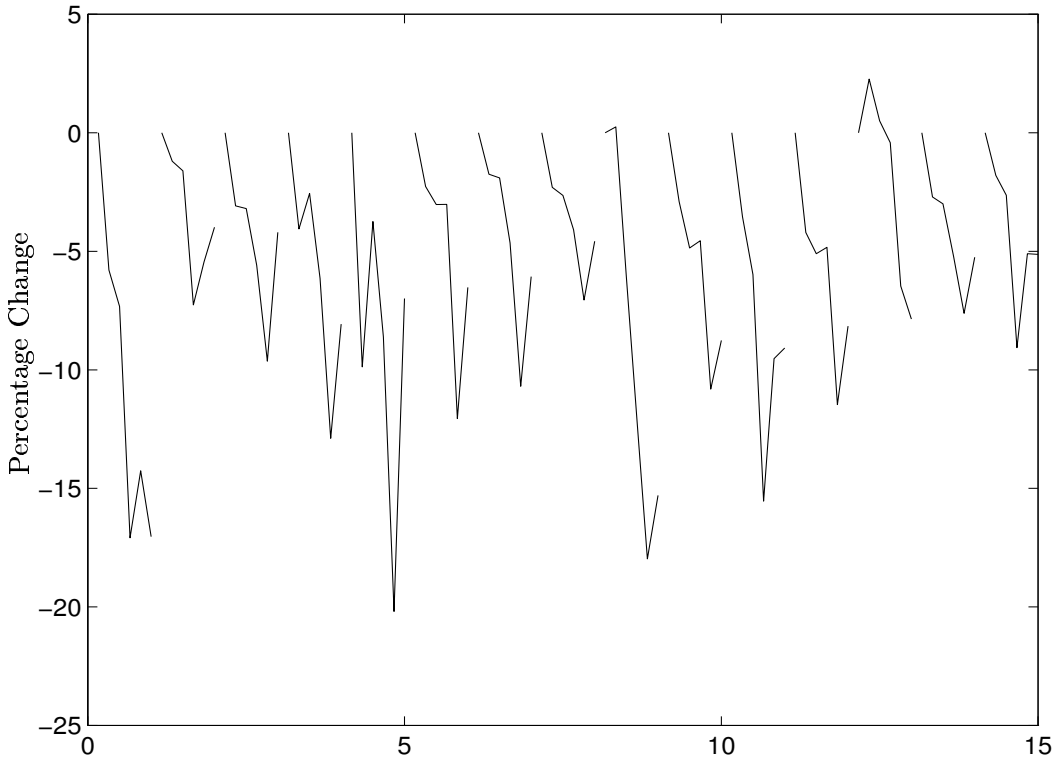
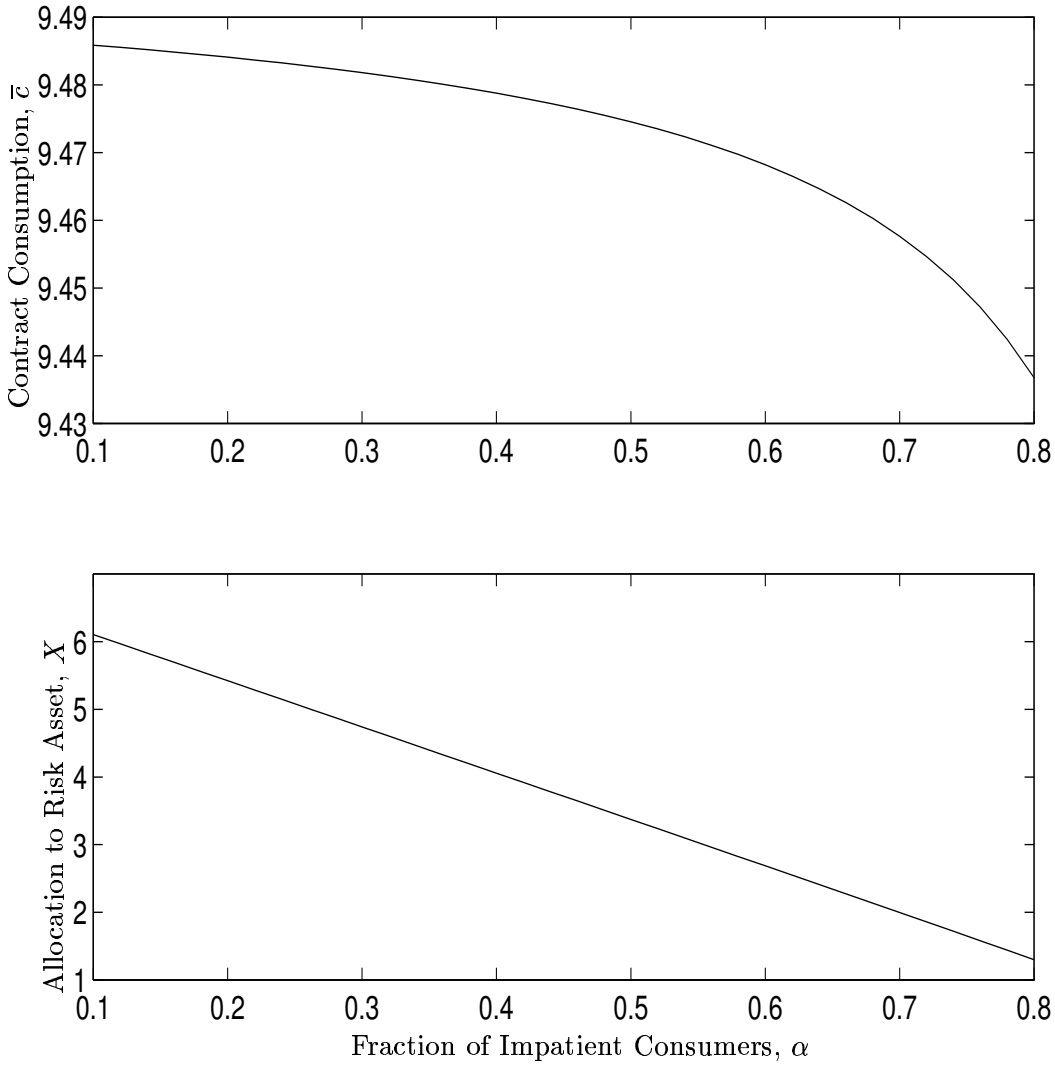
Figure 1
Contagious Effect of Market Crashes



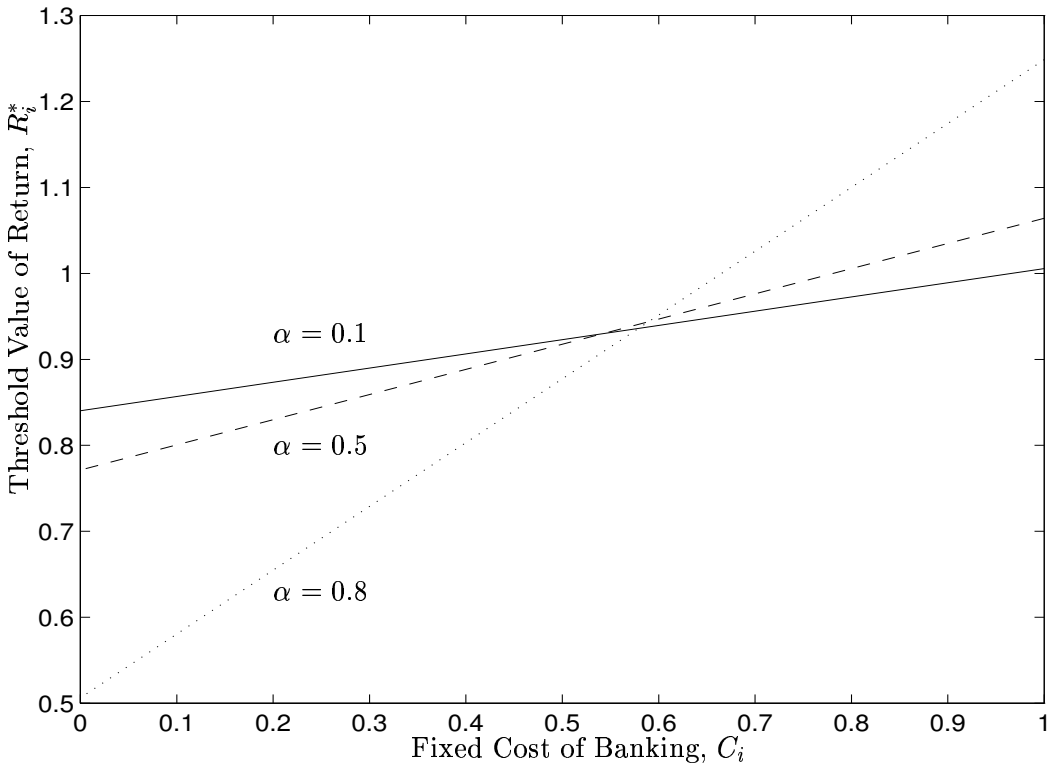
The figures depict the percentage change of the Datastream country price indexes during the week October 22 (Wednesday), 1997 through October 29 (Wednesday), 1997. 15 countries are included. They are, from left to right, Brazil, Canada, France, Germany, Hong Kong, Indonesia, Italy, Japan, Korea, Malaysia, Mexico, Singapore, Taiwan, United Kingdom, and the United States.

Figure 2
Demographic Effect on Optimal Contract and Investment Decision



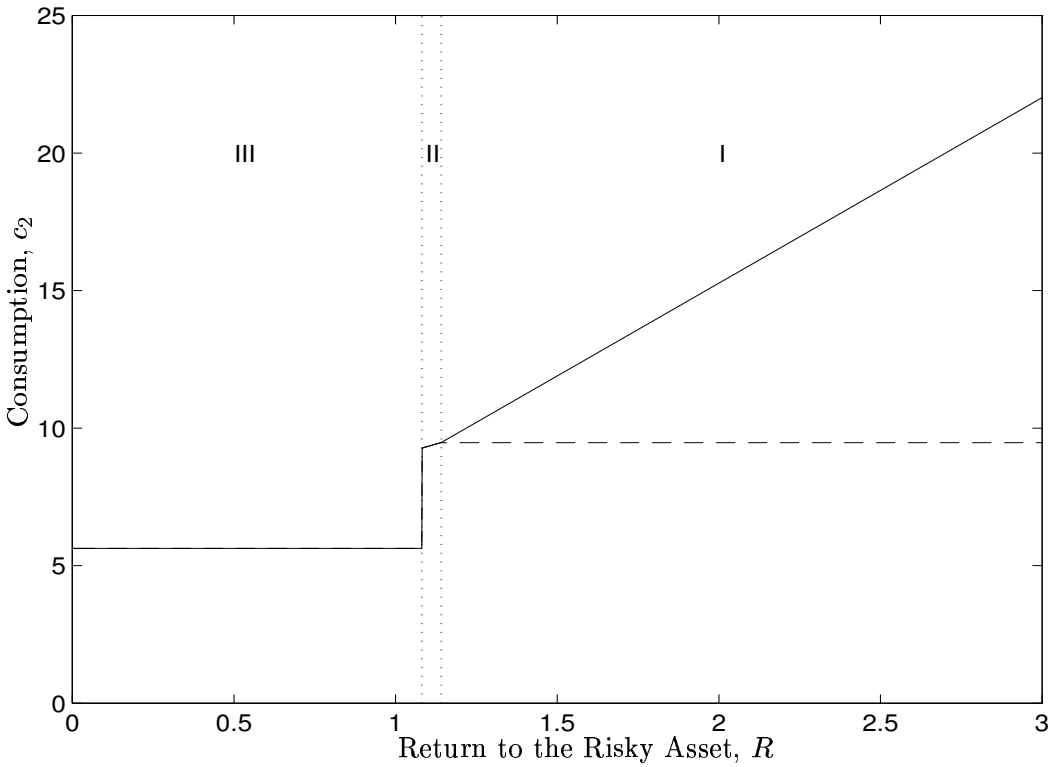
The lines denote the optimal contract ($\overline{c}$) (Above) and optimal asset allocation to the risky asset (X) (Bottom) as a function of the demographic distribution (number of impatient consumers). Log utility is assumed for the consumers. The total endowment is $E = 10$ for each bank. R is uniform distributed between $[0, 3]$. The banking cost C_i uniformly distributed between $[0, 1]$.

Figure 3
Effect of Demographic Composition on the Proneness of Contagion



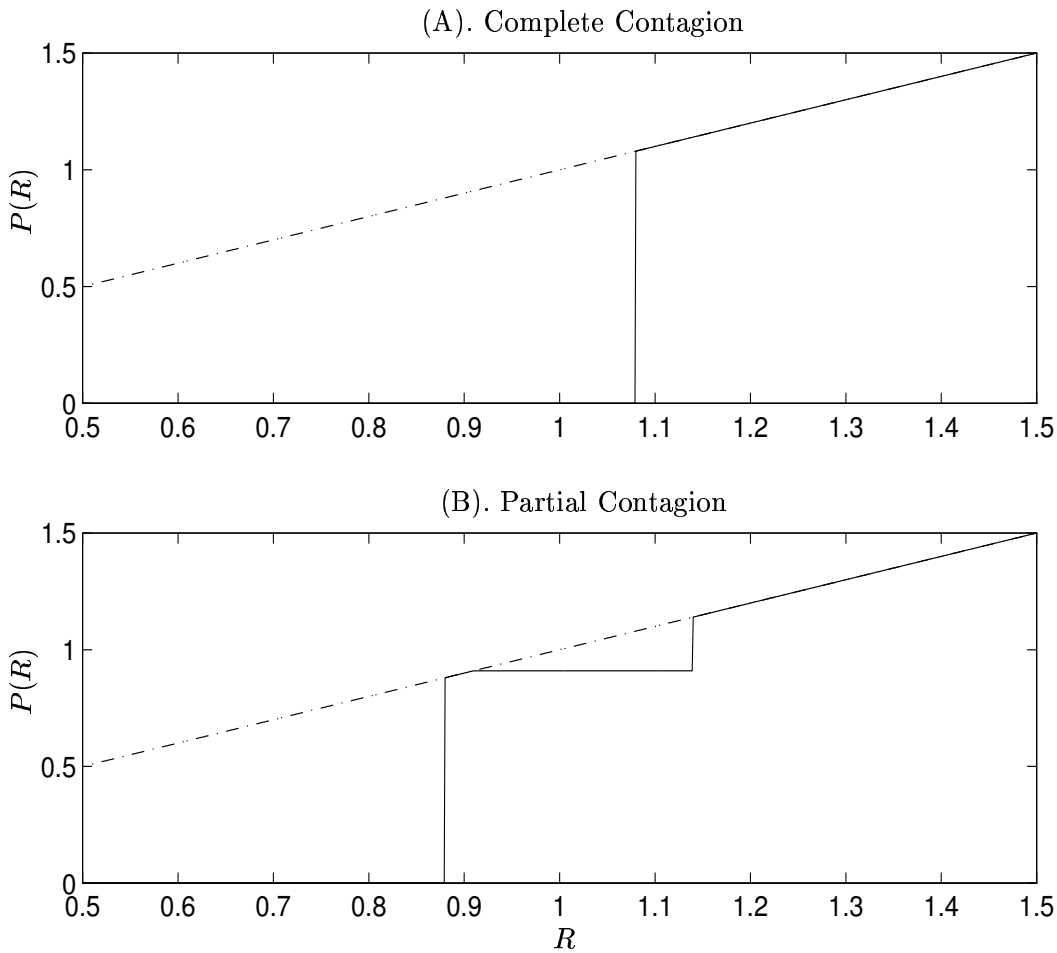
Lines depict the threshold value for bank runs as a function of banking cost. The parameters for the model are: $E = 10$, R is uniform distributed between $[0, 3]$; and C_i uniformly distributed between $[0, 1]$.

Figure 4
Three Phases of An Economy



The solid line denotes the consumption of the patient consumers of the worst bank (bank 1) while the dashed line represents the consumption of the impatient ones of the same bank. Log utility is assumed. The parameters for the model are: $E = 10$, R is uniform distributed between $[0, 3]$; There are altogether six banks with decreasing costs: $C_1 = 1, C_2 = 0.8, ..., C_6 = 0$. The consumption is divided into three phases. In phase I when $R > R_1^*$, all banks are healthy. Early consumers consume $\overline{c}$ as promised while late consumers consume the residual, which is greater than $\overline{c}$. In phase II, $R_2^* < R < R_1^*$, bank 1 goes bankrupt. However, its risky asset can be liquidated at its true value. So both early and late consumers consume split the liquidated value of the firm: $c_1(R) = c_2(R) = L + RX - C_1$. In phase III, when the return on the risky asset $R \leq R_2^*$, a bank run occurs: all banks goes bankrupt. The risky asset cannot be liquidated and thus has zero value. Both early and consumers split the safe asset minus the cost of the bank: $c_1(R) = c_2(R) = L - C_1$.

Figure 5
Equilibrium Market Price for the Risky Asset



Solid lines depicts the equilibrium market price as a function of the realized return for the risky project. The top panel (A) depicts an equilibrium with parameters the same as in Figure 4, where there is either no contagion or complete contagion. The bottom panel (B) depicts an equilibrium economy with partial contagion. The economy has 22 banks with ex post realized banking cost distributed in three groups: The first group has 7 banks with the same banking cost of 1, the second group has one bank with $C_i = 0.9$, and the third group has 14 banks with the banking cost of $C_i = 0.13$. All other parameters are the same as in (A).